Topics from the Theory of Sparse Graphs

Title: MAT 598/494: Topics from the Theory of Sparse Graphs
Semester: Fall 2018
Instructor: Kierstead
Description: We will study the efforts of Nešetřil and Ossona de Mendez to define a theory of sparse graphs. Intuitively a graph class is sparse if none of the subgraphs of its member graphs have too many edges. Some well-known examples are the class of graphs with bounded degree, the class of planar graphs, and the class of graphs with bounded tree-width. These classes have many interesting and useful graph theoretic properties and are particularly amenable to efficient algorithms. The effort to study sparse graphs is motivated in part by the goal of extending these nice results to much broader graph classes. On the other hand, results for broad classes of graphs can often be improved when applied to specific classes. Nešetřil and Ossona de Mendez introduced quite general graph classes with *bounded expansion* that include the examples above as well as the graph classes defined by forbidding a topological minor; even more generally, they defined *nowhere dense* classes. As an example of where this theory leads, we have the following theorem.

Theorem (Grohe, Kreutzer, and Siebertz (2014))**.** *For every nowhere dense class $\mathcal{C}$ and $\varepsilon > 0$, every property of graphs definable in first-order logic can be decided in time $\mathcal{O}(n^{1+\varepsilon})$ on $\mathcal{C}$.*

I will lecture on recent and current papers. Students will be asked to make some of the presentations.

Text: Many recent research papers
Reference: J. Nešetřil and P. Ossona de Mendez, *Sparsity – Graphs, Structures, and Algorithms*. Springer-Verlag, Berlin, Heidelberg, 2012.
Prerequisite: Students should be familiar with basic graph theory, including proof techniques, roughly at the level of MAT 416/513 or higher. Detailed knowledge of specific theorems and proofs is not required, but some familiarity with planar graphs would help.
Audience: This dual undergraduate-graduate course will have combined lectures, but lower standards/expectations for the undergraduates (naturally there are no upper bounds on expectations).

MAT 598 should be useful for all graduate students interested in discrete mathematics. Beginning students will learn about particular research problems of current interest, as well as the process of reading papers and doing mathematical research. The course could form the basis for a summer block grant. More advanced students could use the course to help choose a research topic or for general breadth. Students working in this area can use the course to build a solid foundation to backup their research.

MAT 494 should be interesting and accessible to undergraduates (who meet the prerequisite) from the honors college, computer science, and mathematics. It will provide an introduction to current research in a particular area of graph theory that could provide support for an honors thesis in the current year, or the basis for an NSF Graduate Fellowship application in the following year. More broadly, it will expose students to reading research papers, and the process of mathematical research.