MAT266 Final Exam Review

5.5 Integration by substitution

1. Evaluate $\int \frac{(\ln x)^4}{x}\, dx$.
2. Evaluate $\int \sin(4\pi t)\, dt$.
3. Evaluate $\int x \sin(x^2)\, dx$.
4. Evaluate $\int \frac{\arctan(3x)}{1+9x^2}\, dx$.
5. Evaluate $\int_0^{10} \frac{x}{\sqrt{9+4x}}\, dx$.
6. Evaluate $\int_9^{10} x\sqrt{x-9}\, dx$.
7. In a certain city the temperature in degrees Fahrenheit t hours after 9 am can be modeled by the function

$$T(t) = 50 + 13\sin\left(\frac{\pi t}{12}\right)$$

   (a) What is the *exact* average temperature during the period from 2 pm to 8 pm?

   (b) What is the average temperature during the period from 2 pm to 8 pm rounded to the nearest degree?

6.1 Integration by Parts

8. Evaluate $\int te^{-3t}\, dt$.
9. Evaluate $\int \sqrt{t}\ln t\, dt$.
10. Evaluate $\int_0^{\pi} 7t\sin(19t)\, dt$.
11. Evaluate $\int e^{-x}\cos(6x)\, dx$.
12. Evaluate $\int \ln(7x+1)\, dx$.
13. A patient is given an injection of a drug at a rate of $r(t) = 2te^{-2t}$ ml/sec, where t is the number of seconds since the injection started. What is the amount of the drug injected during the first 10 seconds? *Round your answer to the nearest tenth of a milliliter.*

6.2 Trigonometric Integrals

14. Evaluate $\int \sin^6 x \cos^3 x \, dx$.

15. Evaluate $\int \sin^3(3x) \, dx$.

16. Evaluate $\int \cos^2 \theta \, d\theta$.

17. Evaluate $\int \tan^4 x \sec^4 x \, dx$.

18. Evaluate $\int \frac{x^2}{\sqrt{1-x^2}} \, dx$. *Hint: Let* $x = \sin\theta$

19. Evaluate $\int \frac{x^3}{\sqrt{x^2+9}} \, dx$. *Hint: Let* $x = 3\tan\theta$

6.3 Integration by Partial Fractions

20. Evaluate $\int \frac{1}{(x+6)(x-2)} \, dx$.

21. Evaluate $\int \frac{x-8}{x^2-7x+10} \, dx$.

22. Evaluate $\int \frac{x-13}{(x+8)(x-2)} \, dx$.

23. What is the partial fraction decomposition of $\frac{x^2+1}{(x-4)(x-3)^2}$?

24. What is the partial fraction decomposition of $\frac{3}{x^3+8x^2+16x}$?

6.4 Integration using Tables

25. Evaluate $\int \frac{x}{1+e^{-x^2}} \, dx$ using the formula

$$\int \frac{1}{1+e^u} \, du = u - \ln(1+e^u) + C$$

26. Evaluate $\int \frac{3x^2}{\sqrt{x^6-25}} \, dx$ using the formula

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln|u + \sqrt{u^2 - a^2}| + C.$$

27. Evaluate $\int \ln x \, dx$ using the formula

$$\int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx$$

6.5 Numerical Approximation

28. The record of the speed of a runner during the first 3 seconds of a race is given in the following table:

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (m/s)	0	2.3	5.7	5.9	6.6	7.8	8.9

(a) Use the Trapezoidal Rule with $n = 6$ to approximate the distance the runner covered during these 3 seconds. *Round your answer to the nearest hundredth of a meter.*

(b) Use Simpson's Rule with $n = 6$ to approximate the distance the runner covered during these 3 seconds. *Round your answer to the nearest hundredth of a meter.*

29. Approximate $\int_0^1 \sin(x^2) \, dx$ to 3 decimal places,

(a) using the Trapezoidal Rule with $n = 4$.

(b) using Simpson's Rule with $n = 4$.

30. The time, T, to complete one swing of a pendulum can be modeled by

$$T = 4\sqrt{\frac{L}{g}} \int_0^{\pi/2} \frac{dx}{\sqrt{1 - k^2 \sin^2 x}}$$

where L is the length of the pendulum, g is the acceleration due to gravity, and k is a constant that depends on the maximum angle the pendulum swings. If you use Simpson's rule to approximate T with $n = 16$, what is Δx (the length of each subinterval)?

6.6 Improper Integrals

31. Determine whether $\displaystyle\int_2^3 \frac{13}{\sqrt{3-x}} \, dx$ converges or diverges. If it converges, find its value.

32. Determine whether $\displaystyle\int_e^\infty \frac{79}{x(\ln x)^3} \, dx$ converges or diverges. If it converges, find its value.

33. Determine whether $\int_{-\infty}^{1} \frac{1}{\sqrt{10-x}}\,dx$ converges or diverges. If it converges, find its value.

34. Determine whether $\int_{2}^{\infty} e^{-2x}\,dx$ converges or diverges. If it converges, find its value.

35. Determine whether $\int_{1}^{\infty} \frac{\ln x}{x}\,dx$ converges or diverges. If it converges, find its value.

36. Determine whether $\int_{0}^{1} \frac{1}{x^5}\,dx$ converges or diverges. If it converges, find its value.

37. Suppose the mean life M of a radioactive substance in years is modeled by

$$M = K\int_0^{\infty} e^{\lambda t}\,dt,$$

where K and λ are constants and $\lambda < 0$. What is the mean life of this substance? *Your answer will be in terms of K and λ.*

7.1 Area Between Curves

38. Find the area between the curves $y = 7x - x^2$ and $y = 2x$.

39. Find the area between the curves $x = 3 + 4y^2$ and $x = 7y^2$.

40. Find the area between the curves $y = 8x$, $y = \frac{x}{2}$, and $y = \frac{2}{x}$.

7.2 Volume of Solids of Revolution using Disks/Washers

41. Find the volume of the solid obtained by rotating the region bounded by $x = 6\sqrt{3y}$, $x = 0$, and $y = 3$ around the y-axis using disks/washers.

42. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $x = 0$, $y = 0$, and $x = 1$ around the x-axis using disks/washers.

43. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = x$, $x \geq 0$ around the x-axis using disks/washers.

44. Find the volume of the solid obtained by rotating the region bounded by $y^2 = x$ and $x = 2y$ around the y-axis using disks/washers.

45. What integral represents the volume of the solid obtained by rotating the region bounded by $y = e^x$, $x = 0$, and $y = 3$ around the line $y = 3$ using disks/washers?

46. What integral represents the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 0$, and $x = 2$ around the line $x = 2$ using disks/washers?

47. What integral represents the volume of the solid obtained by rotating the region bounded by $y = \cos x$, $y = 1$, and $x = \frac{\pi}{2}$ around the line $y = 1$ using disks/washers?

7.3 Volume of Solids of Revolution using Shells

48. Find the volume of the solid obtained by rotating the region bounded by $xy = 8$, $x = 0$, $y = 8$, and $y = 10$ around the x-axis using cylindrical shells.

49. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $x = 0$, $y = 0$, and $x = 1$ around the y-axis using cylindrical shells.

50. Find the volume of the solid obtained by rotating the region bounded by $y = 7\sqrt{x}$, $y = 0$, and $x = 1$ around the line $x = -3$ using cylindrical shells.

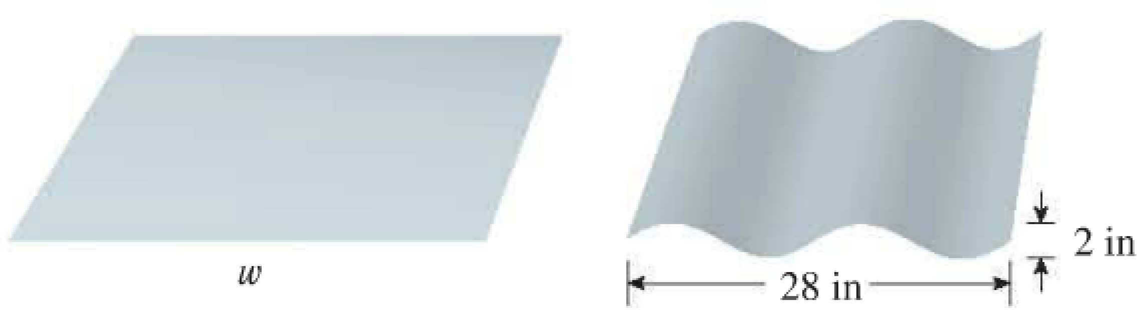
51. What integral represents the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $x = y^2$ around the line $y = -3$ using cylindrical shells?

7.4 Arc Length

52. Find the exact length of the curve $y = 1 + 2x^{3/2}$ for $0 \le x \le 1$.

53. Find the exact length of the curve $x = \frac{y^4}{8} + \frac{1}{4y^2}$ for $1 \le y \le 2$.

54. A manufacturer of corrugated metal roofing wants to produce panels that are 28 inches wide and 2 inches thick by processing flat sheets of metal as shown in the figure. The profile of the roofing takes on the shape of a sine wave that can be modeled by the equation $y = \sin(\pi x/7)$. Set up the integral that would give the width w of a flat metal sheet that is needed to make this 28-inch panel. *Do not solve the integral.*



7.6 Work

55. An inverted circular cone with height 8 meters and base radius 4 meters contains water. Use $1000\,\mathrm{kg/m^3}$ as the density of water and $9.8\,\mathrm{m/s^2}$ for the force of gravity. Set up the integral for the work required to pump:
    (a) all the water out of the tank if the tank is full of water and has no spout
    (b) all the water out of the tank if the tank is filled with water to a depth of 4 meters and has no spout
    (c) all the water out of the tank if the tank is full of water and has a 0.5 meter long spout at the top
    (d) the water in the tank down to a height of 4 meters if the tank is full of water and has no spout

56. A hemispherical tank with radius 2 meters contains water. Use $1000\,\mathrm{kg/m^3}$ as the density of water and $9.8\,\mathrm{m/s^2}$ for the force of gravity. Set up the integral for the work required to pump:
    (a) all the water out of the tank if the tank is full of water and has no spout
    (b) all the water out of the tank if the tank is filled with water to a depth of 1 meter and has no spout
    (c) all the water out of the tank if the tank is full of water and has a 0.5 meter long spout at the top
    (d) the water in the tank down to a height of 1 meters if the tank is full of water and has no spout

57. A cylindrical tank of height 7 feet and base radius 3 feet is filled with water to a depth of 5 feet. Use $62.5\,\mathrm{lb/ft^3}$ as the density of water. Set up the integral for the work required to pump all the water out of the tank if the tank has a 1 foot spout at the top.

8.1 & 8.2 Sequence and Series Convergence

58. Let $a_n = \dfrac{n-1}{8n-1}$.
    (a) Find $\lim\limits_{n\to\infty} a_n$.
    (b) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{8n-1}$ is convergent or divergent. If it is convergent, find its sum.

59. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-7)^{n-1}}{9^n}$ is convergent or divergent. If it is convergent, find its sum.

60. Determine whether the series $\sum_{n=1}^{\infty} \frac{1+9^n}{8^n}$ is convergent or divergent. If it is convergent, find its sum.

61. Determine whether the series $\sum_{n=1}^{\infty} \frac{1+6^n}{7^n}$ is convergent or divergent. If it is convergent, find its sum.

8.4 Limit Ratio Test

62. Describe the behavior of the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.

63. Describe the behavior of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^3 4^n}{n!}$.

64. Describe the behavior of the series $\sum_{n=1}^{\infty} \frac{n!}{3^n}$.

8.5 Power Series Convergence

65. Find the largest open interval of convergence of the series and the radius of convergence $\sum_{n=1}^{\infty} n!(2x-1)^n$.

66. Find the largest open interval of convergence of the series and the radius of convergence $\sum_{n=1}^{\infty} \frac{(x-4)^n}{n9^n}$.

67. Find the largest open interval of convergence of the series and the radius of convergence $\sum_{n=1}^{\infty} \frac{(4x)^n}{n^5}$.

68. Find the largest open interval of convergence of the series and the radius of convergence $\sum_{n=1}^{\infty} \frac{(3x)^n}{n!}$.

8.6 Representing Functions as Power Series

69. Find the power series representation for $f(x) = \frac{x^2}{1-3x^2}$ for $|x| < \frac{1}{\sqrt{x}}$ using the fact that
$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$.

70. Find the power series representation for $f(x) = \dfrac{x^3}{2+x}$ for $|x| < 2$ using the fact that $\displaystyle\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$.

71. Find the power series representation for $f(x) = \dfrac{x^2}{2-5x}$ for $|x| < 2$ using the fact that $\displaystyle\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$.

72. Find the power series representation for $f(x) = \dfrac{1}{\frac{1}{3}+4x}$ for $|x| < \dfrac{1}{12}$ using the fact that $\displaystyle\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$

8.7 Taylor & Maclaurin Series

73. Find the first three nonzero terms of the Taylor Series for $f(x) = \cos x$ centered at $x = \pi$.

74. A function $f(x)$ has a Maclaurin Series on $(-1, 1)$. If possible, find the first three nonzero terms of the Maclaurin series if $f(0) = 2$; $f'(0) = -2$; $f''(0) = 2$.

75. A function $f(x)$ has a Maclaurin Series on $(-1, 1)$. If possible, find the first three nonzero terms of the Maclaurin series if $f(0) = 1$; $f'(0) = 0$; $f''(0) = 1$.

76. Use the known Maclaurin series for $\sin x$ to find the first three **nonzero** terms of the Maclaurin series for $f(x) = x\sin(3x)$.

77. Suppose that the resistivity ρ of a given metal depends on temperature and can be modeled by the equation

$$\rho(t) = \rho_{15} e^{\alpha(2t-30)}$$

where t is the temperature in $^\circ C$, and ρ_{15} and α are constants depending on the metal. At most temperatures, resistivity can be approximated by the second-degree (quadratic) Taylor polynomial centered at $t = 15$. Find an expression for the second-degree Taylor polynomial centered at $t = 15$.

9.1 Parametric Equations

78. Eliminate the parameter to find the Cartesian equation for the curve represented by

$$x = \sqrt{t}, \quad y = 4 - t$$

79. Eliminate the parameter to find the Cartesian equation for the curve represented by

$$x = e^t - 8, \quad y = e^{2t}$$

80. Eliminate the parameter to find the Cartesian equation for the curve represented by

$$x = \sin\left(\frac{\theta}{2}\right), \quad y = \cos\left(\frac{\theta}{2}\right)$$

9.2 Calculus with Parametric Equations

81. For the parametric curve defined by $x = t^3 - t$, $y = t^3 + 4t$, find the points, if any, where the graph has:
   (a) a horizontal tangent line
   (b) a vertical tangent line

82. For the parametric curve defined by $x = t - \frac{1}{t}$, $y = t + \frac{1}{t}$, find the points, if any, where the graph has:
   (a) a horizontal tangent line
   (b) a vertical tangent line

83. Find the equation of the line tangent to the graph of the parametric curve defined by $x = t\cos t$, $y = t\sin t$ for $t = \pi$.

84. If a projectile is fired with an initial velocity of v_0 at an angle of α above the horizontal and air resistance is assumed to be negligible, then its position after t seconds is given by the parametric equations

$$x = (v_0 \cos\alpha)t \qquad y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2$$

   where g is the acceleration due to gravity ($9.8\,\text{m/s}^2$), α is the angle of elevation, and v_0 is the initial velocity.
   (a) Find dy/dx
   (b) Suppose a projectile is fired with $v_0 = 100\,\text{m/s}$ at an angle $\alpha = 60°$. At what time does it reach its maximum height? *Round your answer to the nearest tenth of a second.*

9.3 Polar equations

85. Find a Cartesian equation for the curve described by the polar equation $r = 8cos\theta + 4\sin\theta$.

86. Find a Cartesian equation for the curve described by the polar equation $r = \dfrac{1}{2\cos\theta + 4\sin\theta}$.

87. Find a polar equation for the curve described by the Cartesian equation $x^2 + y^2 = 6x$.

9.4 Arc length & Area in Polar Equations

88. Find the exact arc length of the curve described by the polar equation $r = 6\sin\theta$ for $0 \le \theta \le \pi/2$.

89. Find the exact arc length of the curve described by the polar equation $r = \theta^2$ for $0 \le \theta \le \pi$.

90. Set up an integral that represents the area enclosed by the curve $r = 1 + \cos\theta$.

91. Set up an integral that represents the area enclosed by one petal of the curve $r = 4\cos(3\theta)$.

92. Set up an integral that represents the area outside of the graph of the curve $r = 7$ and inside of the graph of the curve of $r = 7 - 7\sin\theta$.

Answers

1. $\dfrac{1}{5}(\ln x)^5 + C$, where C is a constant

2. $-\dfrac{1}{4\pi}\cos(4\pi t) + C$, where C is a constant

3. $-\dfrac{1}{2}\cos(x^2) + C$, where C is a constant

4. $\dfrac{1}{6}\arctan^2(3x) + C$, where C is a constant

5. $\dfrac{26}{3}$

6. $\dfrac{32}{5}$

7. (a) $50 - \dfrac{26}{\pi}\cos\left(\dfrac{11\pi}{12}\right) + \dfrac{26}{\pi}\cos\left(\dfrac{5\pi}{12}\right)^\circ$ Fahrenheit

(b) 60°Fahrenheit

8. $-\frac{t}{3}e^{-3t} - \frac{1}{9}e^{-3t} + C$, where C is a constant

9. $\frac{2}{3}t^{3/2}\ln t - \frac{4}{9}t^{3/2} + C$, where C is a constant

10. $\frac{7\pi}{19}$

11. $\frac{-e^{-x}\cos(6x) + 6e^{-x}\sin(6x)}{37} + C$, where C is a constant

12. $\frac{(7x+1)\left[\ln(7x+1) - 1\right]}{7} + C$, where C is a constant

13. $0.5\,\text{ml}$

14. $\frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$, where C is a constant

15. $\frac{1}{9}\cos^3(3x) - \frac{1}{3}\cos(3x) + C$, where C is a constant

16. $\frac{\theta}{2} + \frac{\sin(2\theta)}{4} + C$, where C is a constant

17. $\frac{1}{5}\tan^5 x + \frac{1}{7}\tan^7 x + C$, where C is a constant

18. $\frac{1}{2}\left(\arcsin x - x\sqrt{1-x^2}\right) + C$, where C is a constant

19. $\frac{1}{3}(x^2+9)^{3/2} - 9\sqrt{x^2+9} + C$, where C is a constant

20. $-\frac{1}{8}\ln|x+6| + \frac{1}{8}\ln|x-2| + C$ or $\frac{1}{8}\ln\left|\frac{x-2}{x+6}\right| + C$, where C is a constant

21. $2\ln|x-2| - \ln|x-5| + C$, where C is a constant

22. $\frac{21}{10}\ln|x+8| - \frac{11}{10}\ln|x-2| + C$, where C is a constant

23. $\frac{A}{x-4} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$, where A, B, C are constants

24. $\frac{A}{x} + \frac{B}{x+4} + \frac{C}{(x+4)^2}$, where A, B, C are constants

25. $\frac{1}{2}\left(x^2 + \ln(1+e^{-x^2})\right) + C$, where C is a constant

26. $\ln|x^3 + \sqrt{x^6 - 25}| + C$, where C is a constant

27. $x \ln x - x + C$, where C is a constant

28. (a) 16.38 m

    (b) 16.25 m

29. (a) 0.316

    (b) 0.310

30. $\dfrac{\pi}{32}$

31. converges to 26

32. converges to $\dfrac{79}{2}$

33. diverges

34. converges to $\dfrac{1}{2}e^{-4}$

35. diverges

36. diverges

37. $-\dfrac{K}{\lambda}$

38. $\dfrac{125}{6}$

39. 4

40. $4 \ln 2$

41. 486π

42. $\dfrac{\pi}{7}$

43. $\dfrac{4\pi}{21}$

44. $\dfrac{64\pi}{15}$

45. $\displaystyle\int_0^{\ln 3} \pi(3 - e^x)^2 \, dx$

46. $\displaystyle\int_0^{8} \pi(2 - y^{1/3})^2 \, dy$

47. $\int_0^{\pi/2} \pi(1-\cos x)^2\,dx$

48. 32π

49. $\frac{\pi}{5}$

50. $\frac{168\pi}{5}$

51. $\int_0^1 2\pi(3+y)(\sqrt{y}-y^2)\,dy$

52. $\frac{2}{27}(10\sqrt{10}-1)$

53. $\frac{33}{6}$

54. $\int_0^{28} \sqrt{1+\frac{\pi^2}{49}\cos^2\left(\frac{\pi x}{7}\right)}\,dx$

55. (a) $\int_0^8 (9.8)(1000)\pi\left(\frac{y^2}{4}\right)(8-y)\,dy$

    (b) $\int_0^4 (9.8)(1000)\pi\left(\frac{y^2}{4}\right)(8-y)\,dy$

    (c) $\int_0^8 (9.8)(1000)\pi\left(\frac{y^2}{4}\right)(8.5-y)\,dy$

    (d) $\int_4^8 (9.8)(1000)\pi\left(\frac{y^2}{4}\right)(8-y)\,dy$

56. (a) $\int_0^2 (9.8)(1000)\pi(4-(y-2)^2)(2-y)\,dy$

    (b) $\int_0^1 (9.8)(1000)\pi(4-(y-2)^2)(2-y)\,dy$

    (c) $\int_0^2 (9.8)(1000)\pi(4-(y-2)^2)(2.5-y)\,dy$

    (d) $\int_1^2 (9.8)(1000)\pi(4-(y-2)^2)(2-y)\,dy$

57. $\int_0^5 (62.5)(9\pi)(8-y)\,dy$

58. (a) $\frac{1}{8}$

    (b) divergent

59. Converges to $\frac{1}{16}$

60. divergent

61. Converges to $\frac{37}{6}$

62. Absolutely convergent by the limit ratio test

63. Absolutely convergent by the limit ratio test

64. divergent by the limit ratio test

65. Interval of convergence $= \{1/2\}$; radius of convergence $= 0$

66. Largest open interval of convergence $= (-5, 13)$; radius of convergence $= 9$

67. Largest open interval of convergence $= \left(-\frac{1}{4}, \frac{1}{4}\right)$; radius of convergence $= \frac{1}{4}$

68. Largest open interval of convergence $= (-\infty, \infty)$; radius of convergence $= \infty$

69. $\sum_{n=0}^{\infty} 3^n x^{2n+2}$

70. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3}}{2^{n+1}}$

71. $\sum_{n=0}^{\infty} \frac{5^n x^{n+2}}{2^{n+1}}$

72. $\sum_{n=0}^{\infty} 3(-12)^n x^n$

73. $-1 + \frac{1}{2}(x-\pi)^2 - \frac{1}{24}(x-\pi)^4$

74. $2 - 2x + x^2$

75. Not possible

76. $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+2}}{(2n+1)!}$

77. $\rho(t) \approx \rho_{15} + 2\alpha\rho_{15}(t-15) + 2\alpha^2\rho_{15}(t-15)^2$

78. $y = 4 - x^2$ for $x \geq 0$

79. $y = (x+8)^2$ for $x \geq -8$

80. $x^2 + y^2 = 1$

81. (a) none

    (b) $\left(-\frac{2}{3\sqrt{3}}, \frac{13}{3\sqrt{3}}\right)$, $\left(\frac{2}{3\sqrt{3}}, -\frac{13}{3\sqrt{3}}\right)$

82. (a) $(0, -2)$, $(0, 2)$

    (b) none

83. $y = \pi x + \pi^2$

84. (a) $\frac{dy}{dx} = \frac{v_o \sin\alpha - gt}{v_0 \cos\alpha}$

    (b) 8.8 seconds

85. $(x-4)^2 + (y-2)^2 = 20$

86. $y = -\frac{1}{2}x + \frac{1}{4}$

87. $r = 6\cos\theta$

88. 3π

89. $\frac{1}{3}\left((\pi^2+4)^{3/2} - 8\right)$

90. $\int_0^{2\pi} \frac{1}{2}(1+\cos\theta)^2\, d\theta$, or, using symmetry, $\int_0^{\pi} (1+\cos\theta)^2\, d\theta$

91. $\int_{-\pi/6}^{\pi/6} 8\cos^2(3\theta)\, d\theta$, or, using symmetry, $\int_0^{\pi/6} 16\cos^2(3\theta)\, d\theta$

92. $\int_{\pi}^{2\pi} \frac{1}{2}\left((7-7\sin\theta)^2 - 49\right)\, d\theta$, or, using symmetry, $\int_{-\pi/2}^{0} \left((7-7\sin\theta)^2 - 49\right)\, d\theta$