MAT266 Spring 2019 Exam 2 Review

1. Algebraically determine the behavior of $\int_0^\infty 2e^{-x}\,dx$.

2. Algebraically determine the behavior of $\int_0^\infty \frac{1}{4+x^2}\,dx$.

3. Transforms convert one function into another. The Laplace transform of a function $f(t)$ is given by:

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$$

   where s can be treated as a constant.

   (a) Find $\mathcal{L}\{1\}$, $s > 0$.

   (b) Find $\mathcal{L}\{e^{2t}\}$, $s > 2$.

4. Algebraically determine the behavior of $\int_{-\infty}^0 \frac{1}{9+x^2}\,dx$.

5. Algebraically determine the behavior of $\int_{-\infty}^0 \frac{1}{7-2x}\,dx$.

6. Algebraically determine the behavior of $\int_{-\infty}^\infty 3xe^{-x^2}\,dx$.

7. Algebraically determine the behavior of $\int_{-\infty}^\infty \frac{e^{-x}}{1+e^{-2x}}\,dx$.

8. Algebraically determine the behavior of $\int_{-\infty}^\infty \frac{e^{x}}{3-2e^{x}}\,dx$.

9. Algebraically determine the behavior of $\int_2^3 \frac{21}{\sqrt{3-x}}\,dx$.

10. Algebraically determine the behavior of $\int_{-2}^{14} \frac{14}{\sqrt[4]{x+2}}\,dx$.

11. Algebraically determine the behavior of $\int_0^9 \frac{7}{\sqrt[3]{x-1}}\,dx$.

12. What is the area of the region bounded by the curves $y = 5x - x^2$ and $y = 2x$?

13. What is the area of the region bounded by the curves $y = 8\cos(\pi x)$ and $y = 12x^2 - 3$?

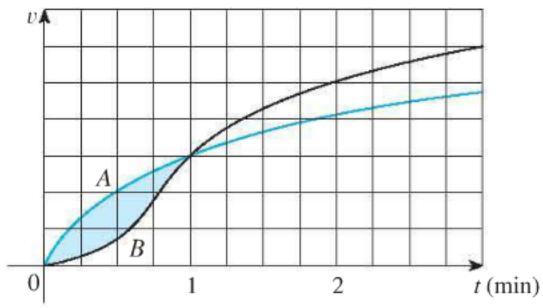
14. What is the area of the region bounded by the curves $x = 9y^2$ and $x = 2 + 7y^2$?

15. What is the area of the region bounded by the curves $y = \frac{2}{x}$, $y = 8x$, and $y = \frac{1}{2}x$, $x > 0$?

16. What is the area of the region bounded by the curves $4x + y^2 = 12$ and $x = y$?

17. Suppose the birth rate of a population is $b(t) = 3700e^{0.05t}$ people per year and the death rate is $d(t) = 1200e^{0.02t}$ people per year. What is the growth in the population over a 20-year period rounded to the nearest person?

18. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.



(a) Which car is ahead at the one minute mark?

(b) What does the shaded region represent?

19. Using the method of disks/washers, what is the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the y-axis?

20. Using the method of disks/washers, what is the volume of the solid obtained by rotating the region bounded by $y = x^2$, $x = 0$, and $y = 4$ about the line $y = 4$?

21. Using the method of disks/washers, what is the volume of the solid obtained by rotating the region bounded by $x = \sqrt{y}$, $x = 3$, and $y = 0$ about the line $x = 3$?

22. Using the method of disks/washers, what is the volume of the solid obtained by rotating the region bounded by $y = x^3$ and $y = x$ for $x \geq 0$ about the y-axis?

23. Using the method of disks/washers, what is the volume of the solid obtained by rotating the region bounded by $y = x^{1/3}$, $x = 8$, and $y = 0$ about the y-axis?

24. Using the method of disks/washers, what is the volume of the solid obtained by rotating the region bounded by $x = \sqrt{y}$, $y = 1$, and $x = 0$ about the x-axis?

25. Using the method of cylindrical shells, what is the volume of the solid obtained by rotating the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis?

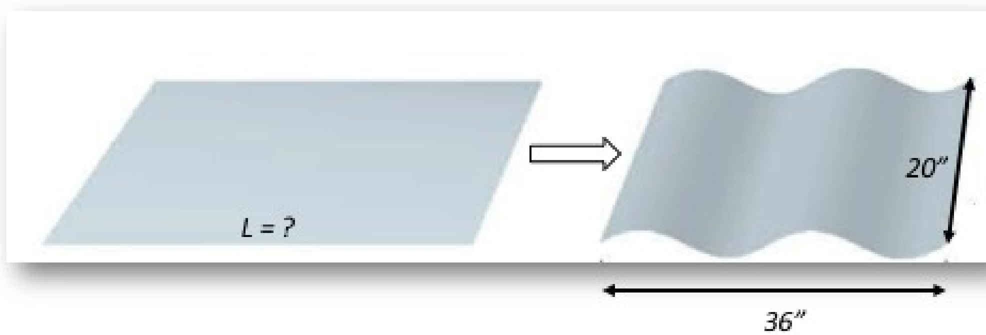
26. Using the method of cylindrical shells, what is the volume of the solid obtained by rotating the region bounded by $y = \sqrt[3]{x}$, $y = 0$, and $x = 1$ about the y-axis?

27. Using the method of cylindrical shells, what is the volume of the solid obtained by rotating the region bounded by $xy = 1$, $x = 0$, $y = 1$, and $y = 3$ about the x-axis?

28. Describe the behavior of:
    (a) the **sequence** $a_n = \dfrac{3n+1}{3n-1}$
    (b) the **series** $\displaystyle\sum_{n=1}^{\infty} \frac{3n+1}{3n-1}$
29. Describe the behavior of:
    (a) the **sequence** $a_n = \dfrac{(3n+1)(-4n+7}{3n^2-1}$
    (b) the **series** $\displaystyle\sum_{n=1}^{\infty} \frac{(3n+1)(-4n+7}{3n^2-1}$
30. Describe the behavior:
    (a) of the **sequence** $a_n = \dfrac{(1+7n)(2n+7)}{2n^2+n}$
    (b) of the **series** $\displaystyle\sum_{n=1}^{\infty} \frac{(1+7n)(2n+7)}{2n^2+n}$
31. Describe the behavior of:
    (a) the **sequence** $a_n = 3 \cdot \left(\dfrac{1}{3}\right)^n$
    (b) the **series** $\displaystyle\sum_{n=1}^{\infty} 3 \cdot \left(\frac{1}{3}\right)^n$
32. Describe the behavior of:
    (a) the **sequence** $a_n = \dfrac{1}{2} \cdot 1^n$
    (b) the **series** $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2} \cdot 1^n$
33. Describe the behavior of
    (a) the **sequence** $a_n = \dfrac{5^{n+2}}{6^{n+2}}$
    (b) the **series** $\displaystyle\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^{n+2}}$
34. Describe the behavior of
    (a) the **sequence** $a_n = \dfrac{3^n+4^n}{7^n}$
    (b) the **series** $\displaystyle\sum_{n=1}^{\infty} \frac{3^n+4^n}{7^n}$
35. Algebraically find the exact arc length of the curve $y = 1 + 6x^{3/2}$, $0 \le x \le 5$. *Do not use your calculator to approximate the answer.*

36. Algebraically find the exact arc length of the curve $y = \frac{1}{3}\sqrt{y}(y-3)$, $1 \le y \le 9$. *Do not use your calculator to approximate the answer.*

37. A manufacturer of corrugated metal roofing wants to produce panels that are 36 inches long and 20 inches wide by processing flat sheets of metal as shown in the figure. The profile of the roofing takes on the shape of a sine wave that can be modeled by the equation $y = \sin(\pi x/9)$. Set up the integral that would give the length L of a flat metal sheet that is needed to make this 36-inch panel. *Do not solve the integral.*



38. Algebraically find the exact arc length of the curve $y^2 = x^3$, $0 \le x \le 4$. *Do not use your calculator to approximate the answer.*

39. What expression represents the length of wire that is needed to fit a curve that is modeled by $y = \cos(\pi x)$ from $0 \le x \le 17$?

40. A drone is traveling due west. The drone's height above ground from horizontal position $x = 0$ to $x = 30$ meters is given by $y = 100 - \frac{1}{20}(x-30)^2$. What expression represents the distance traveled by the drone?

41. Consider a hemispherical tank with a radius of 3 meters that is resting upright on its curved side. Using 9.8 m/s^2 for the acceleration due to gravity and 1,000 kg/m^3 as the

density of water, Set up the integral for the work required to pump the water out of the tank if:

(a) the tank is full of water and it is being pumped out of a 1-meter long vertical spout at the top of the tank.

(b) the tank is half full of water and it is being pumped out of a 1-meter long vertical spout at the top of the tank.

(c) the tank is full of water and it is being pumped out over the top of the tank.

(d) the tank is full of water but you just want to pump half the water out of the tank out over the top of the tank.

42. Consider a cylindrical tank that is standing on its circular base with a radius of 4 meters and a height of 7 meters. Using 9.8 m/s^2 for the acceleration due to gravity and 1,000 kg/m^3 as the density of water, set up the integral for the work required to pump the water out of the tank if:

    (a) the tank is full of water and it is being pumped out of a 0.5-meter long vertical spout at the top of the tank.

    (b) the tank is half full of water and it is being pumped out of a 0.75-meter long vertical spout at the top of the tank.

    (c) the tank is full of water and it is being pumped out over the top of the tank.

    (d) the tank is full of water but you just want to pump half the water out of the tank out over the top of the tank.

43. Consider a tank in the shape of an inverted circular cone with a height of 20 meters and a top radius of 4 meters. Using 9.8 m/s^2 for the acceleration due to gravity and 1,000 kg/m^3 as the density of water, set up the integral for the work required to pump the water out of the tank if:

    (a) the tank is full of water and it is being pumped out of a 1-meter long vertical spout at the top of the tank.

    (b) the tank is half full of water and it is being pumped out of a 0.5-meter long vertical spout at the top of the tank.

    (c) the tank is full of water and it is being pumped out over the top of the tank.

    (d) the tank is full of water but you just want to pump half the water out of the tank out over the top of the tank.

44. Consider a salt water aquarium in the shape of a rectangular prism with a base of length 3 meters, a height of 1.5 meters, and a width of 2 meters. Using 9.8 m/s^2 for the acceleration due to gravity and 1,030 kg/m^3 as the density of salt water, set up the integral for the work required to pump the water out of the tank if:

    (a) the aquarium is full of water and it is being pumped out over the top of the aquarium.

(b) the aquarium is full of water but you just want to pump half the water out of the aquarium over the top of the tank.

45. Evaluate $\int_0^\infty \frac{1}{K^2} e^{\lambda t}\, dt$, where K and λ are constant and $\lambda < 0$. *(Your answer will be a function of both K and λ.)*

ANSWERS

1. 2
2. (a) $\frac{1}{s}$

   (b) $\frac{1}{s-2}$
3. $\frac{\pi}{4}$
4. $\frac{\pi}{6}$
5. Diverges
6. 0
7. $\frac{\pi}{2}$
8. diverges
9. 42
10. Converges to $\frac{448}{3}$
11. Converges to $\frac{63}{2}$
12. $\frac{9}{2}$
13. $\frac{16}{\pi} + 2$
14. $\frac{8}{3}$
15. $4 \ln 2$
16. $\frac{64}{3}$

17. 97,643 people

18. (a) A

    (b) The distance that Car A is ahead of Car B at the 1 minute mark.

19. $\displaystyle\int_0^8 \pi y^{2/3}\,dy = \frac{96\pi}{5}$

20. $\displaystyle\int_0^2 \pi(4-x^2)^2\,dx = \frac{256\pi}{15}$

21. $\displaystyle\int_0^9 \pi(3-\sqrt{y})^2\,dy = \frac{27\pi}{2}$

22. $\displaystyle\int_0^1 \pi(y^{2/3}-y^2)\,dy = \frac{4\pi}{15}$

23. $\displaystyle\int_0^2 \pi(64-y^6)\,dy = \frac{768\pi}{7}$

24. $\displaystyle\int_0^1 \pi(1-x^4)\,dx = \frac{4\pi}{5}$

25. $\displaystyle\int_0^1 2\pi x e^{-x^2}\,dx = \pi - \frac{\pi}{e}$

26. $\displaystyle\int_0^1 2\pi x\sqrt[3]{x}\,dx = \frac{6\pi}{7}$

27. $\displaystyle\int_1^3 2\pi y\frac{1}{y}\,dy = 4\pi$

28. (a) converges to 1

    (b) diverges

29. (a) converges to -4

    (b) diverges

30. (a) converges to 7

    (b) diverges

31. (a) converges to 0

    (b) converges to $\dfrac{3}{2}$

32. (a) converges to $\frac{1}{2}$

    (b) diverges

33. (a) converges to 0

(b) converges to $\frac{125}{36}$

34. (a) converges to 0

    (b) converges to $\frac{25}{12}$

35. $\dfrac{2}{243}(406^{3/2}-1)$

36. $\dfrac{32}{3}$

37. $\displaystyle\int_0^{36}\sqrt{1+\frac{\pi^2}{81}\cos^2\left(\frac{\pi x}{9}\right)}\,dx$

38. $\dfrac{8}{27}(10^{3/2}-1)$

39. $\displaystyle\int_0^{17}\sqrt{1+\pi^2\sin^2(\pi x)}\,dx$

40. $\displaystyle\int_0^{30}\sqrt{1+\frac{1}{100}(x-30)^2}\,dx$

41. (a) $\displaystyle\int_0^{3}(9.8)(1{,}000)\pi(6y-y^2)(4-y)\,dy$

    (b) $\displaystyle\int_0^{1.5}(9.8)(1{,}000)\pi(6y-y^2)(4-y)\,dy$

    (c) $\displaystyle\int_0^{3}(9.8)(1{,}000)\pi(6y-y^2)(3-y)\,dy$

    (d) $\displaystyle\int_{1.5}^{3}(9.8)(1{,}000)\pi(6y-y^2)(3-y)\,dy$

42. (a) $\displaystyle\int_0^{7}(9.8)(1{,}000)16\pi(7.5-y)\,dy$

    (b) $\displaystyle\int_0^{3.5}(9.8)(1{,}000)16\pi(7.75-y)\,dy$

    (c) $\displaystyle\int_0^{7}(9.8)(1{,}000)16\pi(7-y)\,dy$

    (d) $\displaystyle\int_{3.5}^{7}(9.8)(1{,}000)16\pi(7-y)\,dy$

43. (a) $\displaystyle\int_0^{20}(9.8)(1{,}000)\pi\frac{y^2}{25}(21-y)\,dy$

    (b) $\displaystyle\int_0^{10}(9.8)(1{,}000)\pi\frac{y^2}{25}(20.5-y)\,dy$

    (c) $\displaystyle\int_0^{20}(9.8)(1{,}000)\pi\frac{y^2}{25}(20-y)\,dy$

(d) $\displaystyle\int_{10}^{20} (9.8)(1{,}000)\pi\frac{y^2}{25}(20-y)\,dy$

44. (a) $\displaystyle\int_{0}^{1.5} (9.8)(1{,}030)(6)(1.5-y)\,dy$

(b) $\displaystyle\int_{0.75}^{1.5} (9.8)(1{,}030)(6)(1.5-y)\,dy$

45. $-\dfrac{1}{K^2\lambda}$