MAT170 Review Problems for Exam 1

A. Difference Quotient

Find and simplify the difference quotient, $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ for the following functions:

1. $f(x)=\frac{1}{3x}$ 2. $f(x)=-x^2-3x+1$ 3. $f(x)=3x^2-5x+2$ 4. $f(x)=x^3+7x$

B. Inverse Functions

Find the inverse of the following one-to-one functions:

1. $f(x)=\frac{x-3}{x+1}$ 2. $f(x)=7-3x$ 3. $f(x)=x^3-1$ 4. $f(x)=\frac{x}{2x-3}$

5. For question 1 find the domain and range for $f(x)$ and $f^{-1}(x)$.

C. Complex Numbers

Algebraically simplify and write your answer in standard form, $a+bi$, for the following:

1. $f(x)=\frac{3i(2-4i)}{4+3i}$ 2. $f(x)=\frac{-2i(5+3i)}{3+i}$ 3. $f(x)=\frac{8-5i}{3+6i}$

D. Quadratic Functions

Given the following quadratic functions, rewrite the functions in standard form, $f(x)=a(x-h)^2+k$, by completing the square:

1. $f(x)=3x^2-12x+1$
2. $f(x)=-2x^2-16x-40$
3. $f(x)=-3x^2+12x$

E. Transformation of Functions

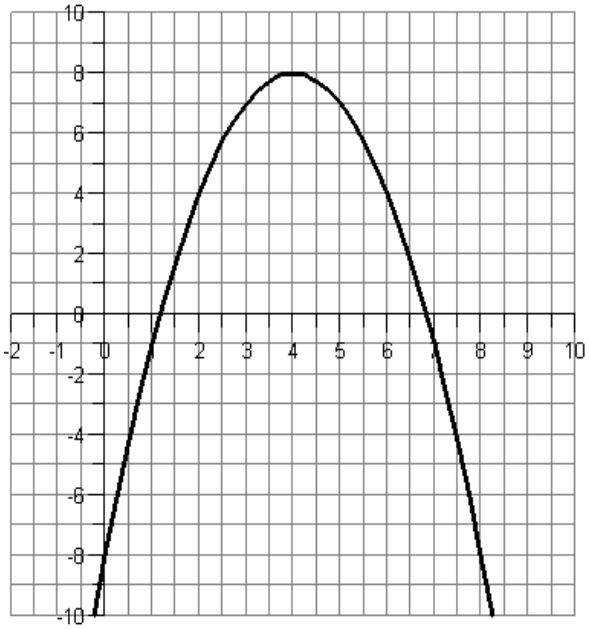
Begin by graphing the standard cubic function, $f(x)=x^3$.

1. After performing the following transformations, find the new function $g(x)$;
   - Shift downward 23 units and shift 13 units to the right
   - Reflect across x-axis, shift upward 5 units and shift 6 units to the left.

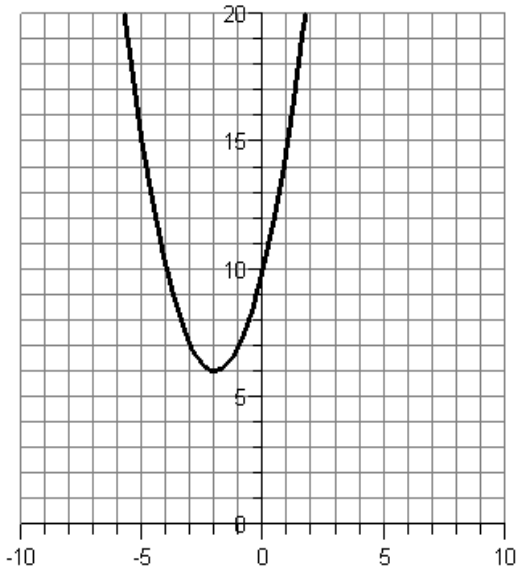
2. Use transformations of the original graph to graph the given function, describe the transformations in words;
   - $g(x)=-(x-1)^3$
   - $h(x)=x^3-2$

3. Using the graphs below, write the formulas for the transformations of $f(x) = x^2$.

a.



b.



F. Combinations and Compositions of Functions

Given the functions:

1. $f(x) = \sqrt{x-3}$ and $g(x) = \dfrac{1}{x}$
2. $f(x) = \dfrac{x}{x+2}$ and $g(x) = 3x - 2$
3. $f(x) = 3x^2 + x - 1$ and $g(x) = 3x - 2$

Use f and g from 1, 2, and 3 to find the following:

a) $(f - g)(x)$ b) $(g \circ f)(x)$

c) $(f \circ g)(x)$ d) $(g \circ g)(x)$

G. Decomposing Functions

Express the given function *h* as a composition of two functions f and g so that $h(x) = (f \circ g)(x)$:

1. $h(x) = (3x^2 - 7x + 1)^6$
2. $h(x) = \sqrt[5]{5x - 23}$

H. Increasing/Decreasing

Graph the following to determine the open intervals where the following functions are increasing and where they are decreasing:

1. $f(x) = \dfrac{x^3}{3} - 4x + 5$ 2. $f(x) = -x^2 + 6x$
3. $f(x) = 0.25x^4 - 0.25x^3 - 4.5x^2 + 4x + 8$

I. Piecewise Functions

Evaluate the following piecewise functions at the given values of the independent variable:

1. $f(x)=\begin{cases}2x+3, & x<0\\ x, & x\geq 0\end{cases}$ Determine $f(0)$

2. $f(x)=\begin{cases}|x-4|, & x\leq 4\\ 5, & x>4\end{cases}$ Determine $f(3)$

3. $f(x)=\begin{cases}\sqrt{4-x}, & x<-3\\ 3x, & x\geq -3\end{cases}$ Determine $f(-2)$

J. Domains of Functions

Find the domain of the following functions:

1. $f(x)=\dfrac{x}{x^2+2x-3}$
2. $f(x)=\dfrac{2x}{x^2-4}$
3. $f(x)=\sqrt{13x+5}$
4. $f(x)=\sqrt{6x+7}-9$
5. $f(x)=2x-\sqrt[3]{13x-7}$

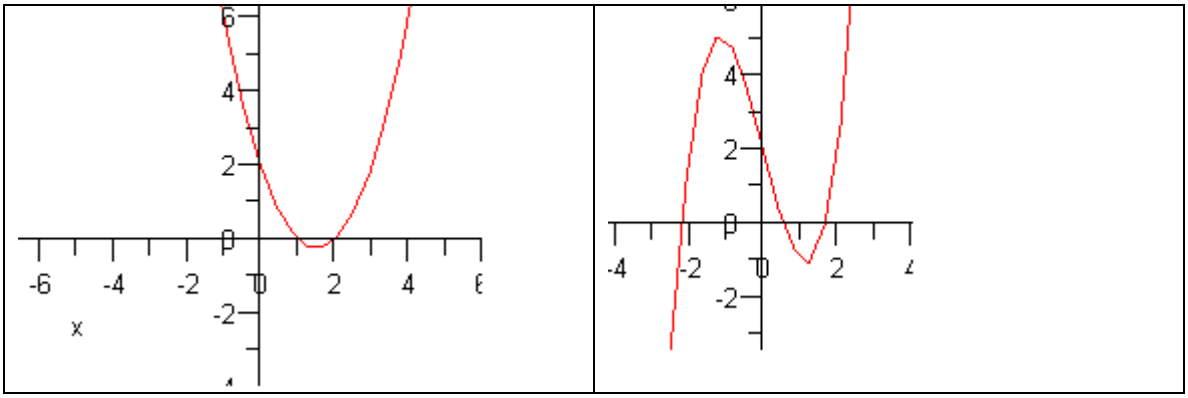
K. Applications of Quadratic Functions

1. A projectile is shot from a 350 foot cliff. The quadratic function, $s(t)$, models the projectile's height above ground, $s(t)=-15t^2+90t+350$, in feet, t seconds after it was shot. When does it reach maximum height? What is the maximum height?

2. An airplane manufacturer can produce up to 15 planes per month. The profit made from the sale of these planes can be modeled by $P(x)=-0.2x^2+4x-3$ where $P(x)$ is the profit in hundred thousand of dollars per month and x is the number of planes made and sold. Based on this model, how many planes should be made and sold to maximize the profit and what is the maximum profit?

L. Finding Values on a Graph

Given the following graphs for f(x), for what values of x does $f(x)=2$?



M. Even/Odd functions

Perform the algebraic tests for even and odd to determine if the function are even, odd or neither.

1. $f(x) = x^5 - 6x^3 + 5x$
2. $f(x) = 7x^2 + 4x + 9$
3. $f(x) = x^4 - 3x^2 + 11$

N. Function behavior

For the function $f(x) = \sqrt{x+4} - 4$, find followings.

1. domain
2. range
3. x-intercept if any as an ordered pair
4. y-intercept if any as an ordered pair

MAT 170 Test 1 Review Answers

There is a reasonable assumption that most of these answers are not incorrect.

A. Difference Quotients: 1. $-\dfrac{1}{3x(x+h)}$ 2. $-2x-h-3$ 3. $6x-5+3h$ 4. $3x^2+7+3xh+h^2$

B. Inverse Functions: 1. $f^{-1}(x) = \dfrac{x+3}{1-x}$ 2. $f^{-1}(x) = \dfrac{7-x}{3}$

3. $f^{-1}(x) = \sqrt[3]{x+1}$ 4. $f^{-1}(x) = \dfrac{3x}{2x-1}$

5. Domain of $f(x) \equiv$ Range of $f^{-1}(x)$ is $(-\infty,-1) \cup (-1,\infty)$.
   Range of $f(x) \equiv$ Domain of $f^{-1}(x)$ is $(-\infty,1) \cup (1,\infty)$.

C. Complex Number: 1. $\dfrac{66}{25} - \dfrac{12}{25}i$ 2. $\dfrac{4}{5} - \dfrac{18}{5}i$ 3. $\dfrac{-2}{15} - \dfrac{7}{5}i$

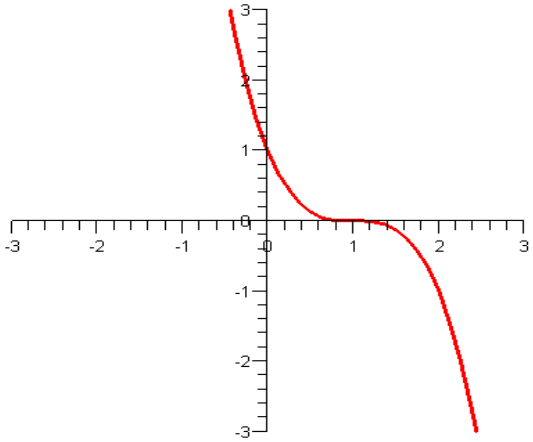
D. Quadratic Functions: 1. $f(x) = 3(x-2)^2 - 11$ 2. $f(x) = -2(x+4)^2 - 8$

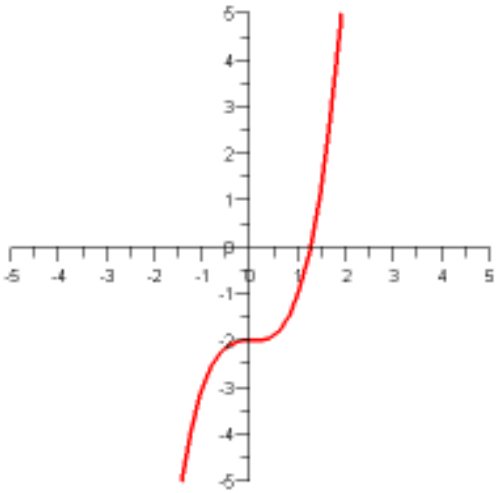
3. $f(x) = -3(x-2)^2 + 12$

E. Transformation of Functions: 1. $g(x) = (x-13)^3 - 23$, $g(x) = -(x+6)^3 + 5$

2. graph of $g(x)$
   reflection across x-axis, right shift 1 unit

   graph of $h(x)$
   downward shift of 2 units





3. a. $g(x) = -(x-4)^2 + 8$ b. $g(x) = (x+2)^2 + 6$

F. Combinations, Compositions: 1. a) $\sqrt{x-3} - \frac{1}{x}$ b) $\frac{1}{\sqrt{x-3}}$ c) $\sqrt{\frac{1}{x} - 3}$ d) x

2. a) $\frac{x}{x+2} - 3x + 2$ b) $\frac{3x}{x+2} - 2$ c) $\frac{3x-2}{3x}$ d) $9x - 8$

3. a) $3x^2 - 2x + 1$ b) $9x^2 + 3x - 5$ c) $27x^2 - 33x + 9$ d) $9x - 8$

G. Decomposing Functions: 1. $f(x) = x^6$, $g(x) = 3x^2 - 7x + 1$

2. $f(x) = \sqrt[5]{x}$, $g(x) = 5x - 23$

H. Increasing, Decreasing: 1. inc. $(-\infty, -2) \cup (2, \infty)$ dec. $(-2, 2)$ 2. inc. $(-\infty, 3)$ dec. $(3, \infty)$

3. inc. $(-2.87, 0.438) \cup (3.18, \infty)$ dec. $(-\infty, -2.87) \cup (0.438, 3.18)$

I. Piecewise Functions: 1. 0 2. 1 3. –6

J. Domains: 1. $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$ 2. $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

3. $\left[\frac{-5}{13}, \infty\right)$ 4. $\left[\frac{-7}{6}, \infty\right)$ 5. $(-\infty, \infty)$

K. Applications (quadratic): 1. 3 seconds , 485 ft. 2. 10 planes, $1,700,000

L. Finding Values: 1. $x = 0, 3$ 2. $x = -2, 0, 2$

M. Even/Odd functions: 1. odd 2. neither 3. even

N. function behavior: 1. domain : $[-4, \infty)$ 2. range : $[-4, \infty)$ 3. x-intercept : $(12, 0)$ 4. y-intercept: $(0, -2)$