## STP226 Review problems Test2 (ch7-8-9)

## Question \#1

In response to increasing weight of airline passengers, the FDA tells airlines to assume that average weight of passengers in the summer (including clothing and carry-on baggage) is 190 pounds with standard deviation of 35 pounds. Weights of passengers are not normally distributed. A commuter plane with maximum passenger weight capacity of 6200 pounds carries 31 passengers. Use Central Limit Theorem to compute approximate probability that their total weight will exceed 6200 pounds, which means that their average weight $\bar{X}$ will be over 200 pounds. Give 4 decimal places for your answer.

$$
P(\bar{X}>200)=
$$

## Use following information in questions \#2-\#3

Diet colas use artificial sweeteners to avoid sugar, but thesesweeteners lose their sweetness over time. Trained testers scored the new cola on a "sweetness score" from 1 to 10 before and after 4 months of storage. Higher score indicates more sweetness. The difference X between the scores (before storageafter storage) was recorded for randomly selected 12 testers. The average change in sweetness was
$\bar{X}=1.02$ with sample standard deviation $\mathrm{s}=0.92$. Assume that $\mathrm{X}=$ change in sweetness is normally distributed with mean $\mu$.

## Question\#2

Obtain $90 \%$ confidence interval for $\mu$, clearly show work by hand.

## Question\#3

The producers of cola claim that their cola on average does not lose sweetness, which means that
$\mu=0$. Is your interval from question 2 supporting that claim or not? Explain your answer.
Select one: Claim is supported Claim is not supported
Explanation:

## Question\#4

Tempe school district administered a standard IQ test to all seven-grade students. Random sample of 30 seven-grade girls in that district had a mean IQ score of 112 points with sample SD of 19 points.
Do we have evidence at $\mathbf{5 \%}$ significance level that mean IQ score of seven -grade girls in that school district exceeds 100 points? Test appropriate hypotheses. Use following parts, clearly show all work.
a) Formulate null and alternative hypotheses:

```
H0: }\mp@subsup{H}{a}{}\mathrm{ :
```

b) Compute the appropriate test statistics.
c)Give the rejection region for your test, include appropriate sketch, marking critical value(s), and clearly label rejection and non-rejection areas.
d)Decide if null hypothesis is rejected or not, explain your decision

Reject $H_{0} \quad$ Do not reject $H_{0} \quad$ (circle one)
Explanation:
e) Clearly answer question posed in the problem. Use complete sentences.

## Question \#5

The level of nitrogen oxides (NOx) in the exhaust of cars of a particular model varies Normally with mean $\mu=0.25$ grams per mile (g/mi) and standard deviation $\quad \sigma=0.08 \mathrm{~g} / \mathrm{mi}$. Government regulations call for NOx emissions no higher than $0.3 \mathrm{~g} / \mathrm{mi}$. A company has 4 cars of this model in its fleet. What is the probability that the average NOx level, $\bar{X}$, of these cars will be below 0.3 $\mathrm{g} / \mathrm{mi}$ limit? Give 4 decimal places for your answer.

$$
P(\bar{X}<0.3)=
$$

## Question \#6

The heights of a certain population of corn plants follow distribution that is not normal, with mean 145 cm and standard deviation of 22 cm . Let $\bar{X}$ represent the mean height of a random sample of 36 plants from the population. Use Central Limit Theorem to compute the probability that $\bar{X}$ will estimate population mean with an error of no more than 10 cm .
$P(135<\bar{X}<155)=$

## Question \#7

The $95.44 \%$ confidence interval for a mean price of all hard cover books in a large book store is (\$23.8, $\$ 30.2$ ). Z-interval procedure was used to compute this interval.
a) What was the sample mean?
b) What is the margin of error?
c) Suppose population standard deviation was $\$ 8$, what was the sample size?

## Question \#8

A random sample of size 15 of delinquent charge accounts of certain large department store has mean of $\$ 58.14$. Assume that the population of of all delinquent charge accounts in that store is nearly normally distributed with no outliers and population standard deviation is $\$ 15.30$.
a. Determine $80 \%$ confidence interval for the actual average size of all delinquent charge accounts at this store (use z-interval procedure)
b. Give interpretation of your interval.
(I) We have confidence that $80 \%$ of all delinquent charge accounts in that store are within the above CI.
(II) We have confidence that $80 \%$ of 15 sampled delinquent charge accounts are within the above CI.
(III) We have $80 \%$ confidence that mean of the 15 sampled delinquent charge accounts is within the above CI.
(IV) We have $80 \%$ confidence that mean of all delinquent charge accounts in that store is within the above CI

## Question \#9

You want to estimate the mean weight of all students in a large university. What should be the size of your sample , so that the $90 \%$ confidence interval for the true population mean weight ( $\mu$ ) will have a margin of error of no more than 5 lb ? Assume that the population standard deviation ( $\sigma$ ) is 20 lb . Assume also normal distribution of all weights of the students at that university.

## Question \#10

Suppose we consider a population of people whose weight is normally distributed with mean $\mu=$ 150 pounds and standard deviation $\sigma=15$ pounds.

Let $\bar{X}$ denote the mean of the sample of size 9 from that population.
a)What type of distribution is the sampling distribution of $\bar{X}$ for samples of size 9 ? Give the mean and the standard deviation of that distribution, use proper symbols.

Sampling distribution of $\bar{X}$ is $\qquad$
$\operatorname{Mean}\left(\mu_{\bar{x}}\right)=$ $\qquad$ St. Deviation $\left(\sigma_{\bar{x}}\right)=$ $\qquad$
b) What percentage of samples of size 9 will have mean ( $\bar{X} \quad$ ) less than 142.5 pounds?
c)What is the probability that, for a sample of size $9, \bar{X}$ will exceed 160 pounds?
d)Compute the probability that, for a sample of size $9, \quad \bar{X}$ will estimate population
mean $\quad \mu \quad$ with an error of no more than 3 lb ?
e) If our population did not have a normal distribution, but highly left skewed distribution, we would not be able to answer questions 1-4 . Explain briefly why not?

## Question \#11

Test scores of all students in Mat 142 classes have a normal distribution. The following is a random sample of 9 test scores from that population:

$$
63,71,80,66,85,92,56,32,77
$$

a. Determine $95 \%$ confidence interval for $\mu$, the true mean test score of all Mat142 students (use t-interval procedure)
b. Based on the interval you obtained in part a, is it likely that $\mu$ is 86 or more? Explain your answer.
Yes
No
(circle one)

Reason: $\qquad$
c. Without computations determine if 99\% CI based on your data will be narrower, wider, or the same width as your CI in part a.

Narrower Wider The same width (circle one)
d. Suppose you computed $90 \%$ CI for $\mu$ using a new sample of size 16. If the margin of of error in your CI was 12 points, what was a sample SD?

## Question \#12

The heights of young women in AZ are normally distributed with mean $\mu$ and standard deviation $\sigma=2.4$ inches. A random sample of 44 AZ women had a sample mean of 68.5 inches. Do we have evidence that $\mu$ is greater than 67 inches? Test appropriate hypothesis, make your conclusions based on the $\mathbf{p}$-value method. Use $\alpha=.01$ ?

Define $\mu$ : $\qquad$
Formulate both hypotheses (use proper symbolic notation)
$\qquad$ $\mathrm{H}_{\mathrm{a}}$ : $\qquad$

Compute appropriate Test Statistics:

Give p-value
( include appropriate sketch)
Decision: $\quad \mathrm{H}_{0}$ rejected
Answer question posed in the problem using a full sentence: $\qquad$

## Question \#13

The owner of a construction company would like to know if his current work crew takes on average less time to build a deck on a house than his past crews. He knows that the average time it took his past crews to build a deck is 28 hours. A random sample of 9 decks that the current crew has built resulted in a sample mean of building time of 24 hours and a sample standard deviation of 6 hours. Conduct the appropriate hypotheses test using the $\alpha=0.05$ and Rejection Region method. Assume that times it takes his current crew to build the decks are normally distributed.

Define $\mu$ :
Formulate both hypotheses (use proper symbolic notation)
$\qquad$
$\mathrm{H}_{0}$ :
$\mathrm{H}_{\mathrm{a}}$ : $\qquad$
Give rejection region :
(draw a sketch and give critical value(s)
Compute appropriate Test Statistics:
Decision: $\mathrm{H}_{0}$ rejected $\quad \mathrm{H}_{0}$ not rejected $\quad$ (select one)

Answer question posed in the problem using a full sentence: $\qquad$

## Question \#14

Suppose you test null hypothesis $H_{0}: \mu=16$ versus alternative $H_{a}: \mu>16$. Answer the following questions:
a)Suppose the test statistics was $\mathrm{z}=1.25$, compute the p-value and decide if $\mathrm{H}_{0}$ be rejected or not at $\alpha=0.05$ ? Include a sketch.

## Sketch:

p-value= $\qquad$

Reject $H_{0}$ Do not reject $H_{0}$ Explain why: $\qquad$
b)Using $\alpha=0.05$, find the rejection region, then decide if there is evidence for alternative hypothesis at $\alpha=0.05$, given that the test statistics was $z=3.14$. Include a sketch.

SKETCH of The REJECTION REGION:
Yes, we have evidence for $H_{a} \quad$ No, we have no evidence for $H_{a}$
Explain why: $\qquad$
c)Suppose p_value for the test was .034 , would you reject $H_{0}$ at $5 \%$ significance level? Explain why.
$\qquad$
d)Suppose that the p-value for the right tailed test was .034 , what will be the p-value if we change alternative hypothesis to $H_{a}: \mu \neq 16$ ?

P-Value: $\qquad$
e)Suppose we rejected null hypothesis and later we find out that true population mean was 16. Was our decision to reject correct? If not, what type of error was made?

CORRECT TYPE I ERROR TYPE II ERROR (select one)
f) Suppose $95 \%$ CI for $\mu$ was (14.6, 17.8). Based on that interval, would null hypothesis $H_{0}: \mu=16$ be rejected in favor of $H_{a}: \mu \neq 16$ ?
$\qquad$
g)Suppose $95 \%$ CI for $\mu$ was (12.6, 15.1), would you reject null hypothesis $H_{0}: \mu=16$ in favor of $H_{a}: \mu \neq 16$ based on that CI?

YES NO Reason: $\qquad$

## Multiple choice questions. Select one from A to E as appropriate.

## Use following for Questions \#15-\#19

## Student Study Times.

A survey asked the following question: " About how much time (in minutes) do you study on a typical weeknight?" of randomly selected 46 first-year ASU students. The sample mean was 118 minutes. Let $\mu$ be the mean study time of all first-year ASU students, assume that population standard deviation $\sigma=65$ minutes.

## Question \#15

Compute a margin of error in a $90 \%$ confidence interval for $\mu$. Give 2 decimal places for the final answer.
A) 19.17
B) 18.78
C) 15.77
D)19.30
E) none of these

## Question \#16

What sample size is needed for a margin of error in $95.44 \%$ confidence interval for $\mu$ to be no more than 5 minutes.
A) $n \geq 650$
B) $n \geq 458$
C) $n \geq 676$
D) $n \geq 277$
E) none of these

## Question \#17

Suppose you want to test if $\mu$ is less than 2 hours (120 minutes). Formulate null and alternative hypotheses to be tested:
A) $H_{0}: \mu \geq 118$
B) $H_{0}: \mu<120$
C) $H_{0}: \mu=118$
D) $H_{0}: \mu=120$
$H_{a}: \mu=120$
$H_{a}: \mu=118$
$H_{a}: \mu<118$
$H_{a}: \mu<120$
E) none of these

## Question\#18:

Suppose the p-value for your test you conducted in previous question (\#8) was 0.42 , what is the conclusion for your test at $5 \%$ significance level? Select one of the answers below:
(A) Reject $H_{0}$, we have no evidence for $H_{a}$ at $\alpha=0.05$
(B) Do not reject $H_{0}$, we have no evidence for $H_{a}$ at $\alpha=0.05$
(C) Reject $\quad H_{0}$, we have evidence for $H_{a}$ at $\alpha=0.05$
(D) Do not reject $H_{0}$, we have evidence for $H_{a}$ at $\alpha=0.05$

## Question\#19

Suppose you want to test the following hypotheses: $\mathrm{H}_{0}: \quad \mu=110$ versus $\mathrm{H}_{\mathrm{a}}: \quad \mu \neq 110$ at $5 \%$ significance level. Select critical value(s) for the rejection region for your test:
(A) $\pm 1.645$
(B) $\pm 2.014$
(C) $\pm 1.28$
(D) $\pm 1.96$
(E) none of these

## Use following information in Questions \#20- \#22:

## Cholesterol Levels of young males.

Suppose a researcher wants to test if a mean cholesterol level ( $\mu$ ) of young males who experienced a mild heart attack is higher than that of healthy young males, which is known to be $188 \mathrm{mg} / \mathrm{dl}$. His hypotheses are: $\mathrm{H}_{0}: \mu=188$ versus $\mathrm{H}_{\mathrm{a}}: \mu>188$

## Question\#20

Suppose he knows the population standard deviation and is using a z-test, and the test statistics he received is $\mathrm{z}=2.37$. Compute the $\mathbf{p}$ - value for his test. Give $\mathbf{4}$ decimal places.
A)0.0089
B) 0.9911
C) 0.0178
D) 1.9822
(E) none of these

## Question\#21

Suppose he does not know the population standard deviation and is using a t-test , his sample size $\mathrm{n}=$ 15 and and the test statistics he received is $t=1.90$. Estimate the $\mathbf{p}$ - value for his test from the t -table:
A) $0.025<p-$ value $<0.05$
B) $0.05<p-$ value $<0.025$
C) $0.05<p-$ value $<0.10$
D) $0.01<p-$ value $<0.025$
(E) none of these

## Question\#22

Suppose null hypothesis was rejected at 5\% significance level, only one of the following is the correct conclusion for our hypotheses test, circle the correct answer.
A) We have no evidence at $5 \%$ significance level that mean cholesterol level for young males that experienced a mild heart attack is higher than mean cholesterol level for healthy young males
B) We have evidence at 5\% significance level that mean cholesterol level for young males that experienced a mild heart attack is higher than mean cholesterol level for healthy young males.
C) We have evidence at $5 \%$ significance level that mean cholesterol level for young males that experienced a mild heart attack is lower than mean cholesterol level for healthy young males
D) We have evidence at 5\% significance level that mean cholesterol level for young males that experienced a mild heart attack is the same as mean cholesterol level for healthy young males

## Question\#23

Suppose the mean annual income for adult women in one city is $\$ 28,520$ with standard deviation of $\$ 5190$ and the distribution is left skewed. What is the sampling distribution of the sample mean $\bar{x}$ for samples of size 49
A) approximately normal distribution
B) $t$ distribution with 48 degrees of freedom
C) standard normal distribution
D) can't specify, because sample is not large enough

## TRUE- FALSE questions. Decide if each of the following statements is True or False.

Statement\#1 Suppose 90\% confidence interval for a mean age of participants in a large mathematical conference, based on a random sample of 120 participants, is ( 35,49 ). We can say that $90 \%$ of all participants in that mathematical conference are between 35 and 49 years old.

## True False

Statement\#2 Suppose the mean annual income for adult women in one city is $\$ 28,520$ with standard deviation of $\$ 5190$ and the distribution is extremely right skewed. For the samples of size 19, sample mean has approximately normal distribution.

## True False

Statement\#3 If we compute 95\% and 90\% confidence intervals for the mean final exam score of all Mat 117 students at ASU last semester, then $95 \%$ confidence interval will be wider than $90 \%$ confidence interval.

## True False

Statement\#4 In testing hypothesis null hypothesis will be rejected if p -value for the test is greater than selected significance level $\alpha$.

## True False

Statement\#5 If we test $\mathrm{H}_{0}: \mu=18$ versus $\mathrm{H}_{\mathrm{a}}: \mu \neq 18$ and $\mathrm{H}_{0}$ is rejected, but later a mega study concluded that $\mu=22$, then by rejecting $\mathrm{H}_{0}$ we committed Type I error.

## True False

Statement\#6 If test statistics falls outside of the rejection region, we reject null hypothesis.

## True False

Statement\#7 Suppose we conducted a z-test of the following hypotheses: $\mathrm{H}_{0}: \mu=8$ versus $\mathrm{H}_{\mathrm{a}}: \mu \neq 8$ and we rejected null hypothesis at $5 \%$ significance level. In that case $95 \%$ confidence interval for $\mu$ computed from the same data would contain 8 .

## True False

Statement\#8 If p -value for the right tailed t -test test is 0.032 , then p -value for the two tailed t test is 0.064 .

True False

Key.
Question \#1
$\bar{x}$ has approximately Normal distr. with mean 190 lb and SD of 6.286 lb , so $P(\bar{x}>200)=P(z>1.59)=0.0559$

## Question \#2

T interval, df=11 $\quad t_{0.05}=1.796 \quad E=1.796\left(\frac{0.92}{\sqrt{12}}\right)=0.48 \quad 1.02 \pm 0.48$ gives $(0.54,1.50)$

## Question \#3

Claim is not supported, CI is above 0 and we have $90 \%$ confidence that $\mu$ is inside that interval.

## Question \#4

a) $H_{0}: \quad \mu=100 \quad H_{a}: \quad \mu>100$
b) $t=3.46$
c) $\mathrm{df}=29, \mathrm{cv}=1.699$, sketch must show t-curve, are right of cv is a rejection region (inclusive)
d)Reject $H_{0}$

Explanation: 3.46 falls into the rejection region
e) At 5\% significance level data presents evidence that mean IQ score for seven-grade girls in that school district exceeds 100 points.

## Question \#5

$\bar{X}$ has Normal distr. with mean $0.25 \mathrm{~g} / \mathrm{mi}$ and SD of $0.04 \mathrm{~g} / \mathrm{mi}$, so

$$
P(\bar{X}<0.3)=P(z<1.25)=0.8944
$$

## Question \#6

$\bar{X}$ has approximately Normal distr. with mean 145 and SD of 3.67 cm , so
$P(135<\bar{X}<155)=P(-2.72<z<2.72)=0.9935$

## Question \#7

a) $\bar{x}=(30.2+23.8) / 2=27$
b) $\mathrm{E}=(30.2-23.7) / 2=3.2$
c) $E=3.2=2 \frac{8}{\sqrt{n}}$ from this $\mathrm{n}=(16 / 3.2)^{2}=25$

Question \#8
a) z-interval $\quad E=1.28\left(\frac{15.3}{\sqrt{15}}\right)=5.06 \quad 58.14 \pm 5.06$ gives $(53.08,63.2)$
b) IV

## Question \#9

$$
n=\left(\frac{1.645(20)}{5}\right)^{2}=43.3 \text {, so } \quad n \geq 44
$$

## Question \#10

a) Sampling distribution of $\bar{X}$ is normal

Mean $\left(\mu_{\bar{x}}\right)=150$ St. Deviation $\left(\sigma_{\bar{x}}\right)=15 / 3=5$
b) $z=\frac{(142.5-150)}{5}=-1.5 \quad 6.68 \%$
c) $P(\bar{x}>160)=P(z>2)=0.0228$
d) $P(147 \leq \bar{x} \leq 153)=P(-0.6 \leq z \leq 0.6)=.4515$
e) No, we would need a large sample, at least 30 .

## Question \#11

a) $\bar{x}=69.11$ and $\mathrm{s}=17.88 \mathrm{CI}:(55.4,82.9)$
b) No, CI is below 86
c) Wider, $t$-value will be larger
d) $\mathrm{df}=15, \quad t_{0.05}=1.753 \quad s=\frac{12 \sqrt{16}}{1.753}=27.4$

## Question \#12

$\mu=$ mean height of young women in AZ
$H_{0}: \mu=67 \quad H_{a} \quad \mu>67$
$\mathrm{z}=4.15 \mathrm{p}$-value $=0$ (from tables) sketch : area right of 4.15 under $\mathrm{N}(0,1)$
$H_{0}$ rejected
There is evidence at $1 \%$ sign. level that mean height of young women in AZ is greater than 67 inches

## Question \#13

$\mu=$ mean time to build the deck for the new crew.
$H_{0}: \mu=28 \quad H_{a} \quad \mu<28$
$\mathrm{t}=-2 \mathrm{cv}=-1.86$, sketch :Rejection region left of cv under t curve with $\mathrm{df}=8$
$H_{0}$ rejected
There is evidence at $5 \%$ sign. level that mean time the new crew takes to build the deck is lower than 28 hours.

## Question \#14

a. $\mathrm{pv}=.1056$ (area right of 1.25), $H_{0}$ not rejected, since pv>. 05
b. Rejection region=area right of 1.645, $H_{0}$ rejected, so we have evidence for $H_{a}$
c. $0.034<0.05$, yes
d. $p v=2(0.034)=0.068$
e. Type I error
f. no, 16 inside CI
g. yes, 16 outside CI

Questions 15-23
C C D B D A A A
Questions (Statements) 1-8
FFTFTFTT

