MAT117 Review for Final Exam

Domain of functions

1. Find the domain of the following functions:

a. $f(x)=\sqrt{3x-12}$ b. $f(x)=\sqrt[4]{3x+15}$

2. Find the domain of the following functions:

a. $f(x)=\dfrac{2x+3}{x^2+5x}$ b. $f(x)=\dfrac{x}{x^2-25}$

3. Find the domain of the following functions:

a. $f(x)=\log_2(2x-7)$ b. $f(x)=\ln(4+5x)$

Difference quotient

4. Find and simplify the difference quotient, $\dfrac{f(x+h)-f(x)}{h}$, $h\neq 0$ for the function $f(x)=3x-x^2$

5. Find and simplify the difference quotient, $\dfrac{f(x+h)-f(x)}{h}$, $h\neq 0$, for the function, $f(x)=\dfrac{4}{x}$.

Average rate of change

6. Find the average rate of change for $f(x)=x^2-3x+2$ from x = -1 to x = 4.

7. Find the average rate of change for $f(x)=3-x^2$ from $x=-1$ to x = 3.

Transformations

8. Find the function, $g(x)$, that is finally graphed after the following transformation are applied to the graph of $y=x^2$, but is shifted down 1 unit, reflected across the x-axis, and finally shifted right 3 units.

9. Find the function, $g(x)$ that is finally graphed after the following transformation are applied to the graph of $y=x^2$, but is shifted up 1 unit, reflected across the x-axis, and finally shifted left 2 units.

Linear functions and their applications

10. A truck rental company rents a moving truck one day by charging \$35 plus \$0.05 per mile. Write a linear equation that relates the cost C, in dollars, of renting the truck to x, the number of miles driven. What is the cost of renting the truck is driven 180 miles?

11. The Monthly cost C, in dollars, for international calls on a certain cellular phone plan is given by the function $C(x)=0.38x+5$, where x is the number of minutes used. What is the cost if you talk on the phone for x=50 minutes?

Linear Regression

12. The table shows (lifetime) peptic ulcer rates (per 100 population) for various education levels as reported by the 1989 National Heath Interview Survey.

Educational Level (Year), t	8	11	12	15
Ulcer Rates, R	17	15.5	12	10

Using a graphing calculator, find the regression line and use that line to predict the ulcer rate in 17 years.

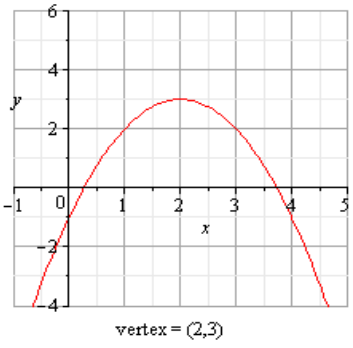
Application of Quadratic Function/ max & min

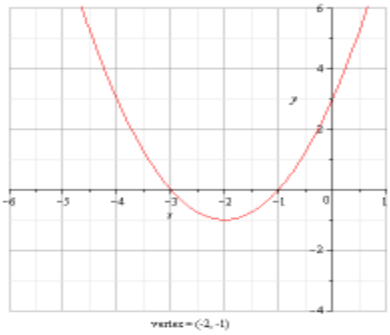
13. A developer wants to enclose a rectangular grassy lot that borders a city street for parking. If the developer has 350 feet of fencing and does not fence the side along the street, what is the largest area that can be enclosed?

14. The price p and the quantity x sold of a certain product obey the demand equation: $p(x) = -\frac{1}{5}x + 300$.

How many units x should be sold to maximize revenue.

15. Given the graphs of the quadratic functions, determine their definition (formula) from their graph.

a. 

b. 

Vertical and Horizontal asymptotes / applications

16. Find the equation of the vertical asymptotes of the following functions:

a. $f(x) = \dfrac{1-x^2}{x^2-5x+6}$. b. $g(x) = \dfrac{x^2-4}{3x^2-5x-2}$.

17. Find the equation of the horizontal asymptotes of the following functions:

a. $f(x) = \dfrac{1-x^2}{3x^2-5x+6}$. b. $g(x) = \dfrac{9x^2+x+12}{3x^2-5x-2}$.

18. The population P for an insect t months after being transplanted is $P(t) = \dfrac{50(1+0.4t)}{(2+0.01t)}$. Determine the horizontal asymptote of P(t). Describe this in the context of the problem.

19. A farmer introduces 100 trout into his pond. The population of the trout growth can be modeled by the function $p(t) = \dfrac{150t+100}{0.04t+1}$, where t is time measured in months

Determine the horizontal asymptote of p(t). Describe this in the context of the problem.

Rational zero test

20. Find a polynomial with zeros at 3 multiplicity 2; –1 multiplicity 2; degree 4; leading coefficient 1.

21. Find a polynomial with zeros at 1 multiplicity 2; $-\frac{1}{2}$ multiplicity 2; degree 4; leading coefficient 1.

Find all the zeros / complex zeros

22. Suppose $x = -6$ is a zero of $p(x) = 2x^3 + 11x^2 - 7x - 6$ Find the rest of the real zeros to factor $f(x)$ completely.
23. Suppose $x = -3$ is a zero of $p(x) = x^3 + 3x^2 + x + 3$ Find the rest of the real zeros to factor $f(x)$ completely.

Composite functions

24. Let $f(x) = x^2 - 4x$, and $g(x) = 3x - 1$, find the following:
a. $(f \circ g)(x)$. b. $(g \circ f)(x)$.

25. Let $f(x) = x^2 + 2$; $x \geq 0$ and $g(x) = \sqrt{x-2}$, find the following:
a. $(f \circ g)(x)$. b. $(g \circ f)(x)$.

Inverse functions

26. Find the inverse function, $f^{-1}(x)$, given the one-to-one function $f(x) = \dfrac{2x+3}{x-1}$.

27. Find the inverse function, $g^{-1}(x)$, given the one-to-one function $g(x) = 4x^3 - 3$.

Compress to a single logarithm

28. Express as a single logarithm: $\log(x) - \log(x^2 - 9) - \log(2) + \log(x - 3)$.
29. Express as a single logarithm: $\ln(x^2 - 1) - 2\ln(x + 1)$.

Exponential equations

30. Solve the equations.
a $3^{2x} - 13(3^x) + 42 = 0$ b. $7^{2x} + 2(7^x) - 8 = 0$

Logarithmic equations

31. Solve the equations:
a $\ln(x+5) - \ln(17 - 3x) = 0$ b. $\log_3(x) + \log_3(x - 8) = 2$

Compound Interest, $A = P(1+\frac{r}{n})^{nt}$, $A = Pe^{rt}$

32. How long does it take \$1500 to double if it is invested at 4% interest, compounded monthly?
33. How long does it take \$1500 to double if it is invested at 4% interest, compounded continuously?

Exponential growth & decay,

34. A certain radioactive isotope has a half-life of 1500 years. How many years would be required for a given amount of this isotope to decay to 35% of that amount?

35. A piece of charcoal is found to contain 40% of the carbon-14 that it originally had. How old is the piece of charcoal? The half-life of carbon-14 is 5600 years.

Systems of equations in 2 variables

36. Solve the system of equations(or if it can't be solved say inconsistent), $\begin{cases} 7x-5y=15 \\ x+5y=1 \end{cases}$.

37. Solve the system of equations(or if it can't be solved say inconsistent), $\begin{cases} 8x-3y=5 \\ 16x-6y=7 \end{cases}$.

Application of system of equations

38. At Sunda's Snacks, caramel corn worth \$2.50 per pound is mixed with honey roasted nuts worth \$7.50 per pound in order to get 20 lb of mixture worth \$4.50 per pound. How much of each snack is used?

39. A movie theater charges \$9 for adults and \$7 for senior citizens. On a day when 325 people paid admission, the total receipts were \$2495. How many people who paid were adults? How many were senior citizens?

Writing several terms of a sequence

40. Write the first three terms of the following sequences:

a. $a_n = \dfrac{(-1)^{n+1} \cdot n}{(n+3)^2}$ b. $a_n = \dfrac{2^n}{3^n+1}$

Recursive formula

41. Find the 2nd and 3rd term of the given recursively defined sequence $a_n = 4a_{(n-1)}+3n$ *and* $a_1 = 5$
42. Find the 2nd and 3rd term of the given recursively defined sequence $a_n = n+3a_{(n-1)}$ *and* $a_1 = -2$

Write down the nth term.

43. Write down the n^{th} term of the sequence $\{a_n\}$ suggested by the pattern $1, \frac{-1}{4}, \frac{1}{9}, \frac{-1}{16}, \frac{1}{25}, \ldots$

44. Write down the n^{th} term of the sequence $\{a_n\}$ suggested by the pattern $\frac{5}{3}, \frac{6}{9}, \frac{7}{27}, \frac{8}{81}, \ldots$

Summation Notation

45. Express the sum, $\frac{1}{7}+\frac{3}{8}+\frac{5}{9}+\frac{7}{10}, \ldots + \frac{2(16)-1}{22}$ using summation notation.

46. Express the sum, $\frac{2}{3}-\frac{4}{9}+\frac{8}{27}-\ldots+(-1)^{12}(\frac{2}{3})^{11}$ using summation notation.

Find a sum using sigma notation / application

47. Using the properties of sequences and the summation formulas of sequence find the sum of $\sum_{k=1}^{30}(5k+3)$.

48. Using the properties of sequences and the summation formulas of sequence find the sum of $\sum_{k=1}^{26}(3k-7)$.

Determine if a sequence is an Arithmetic sequences

49. Find the 25^{th} term of the arithmetic sequence 17, 22, 27, …

50. Find the 20^{th} term of the arithmetic sequence 5, -2, -9, …

Write general term of arithmetic sequence

51. Find the first term, a_1, and the common difference, *d*, of the arithmetic sequence if the 4^{th} term is 8 and the 20^{th} term is –64.

52. Find the first term, a_1, and the common difference, *d*, of the arithmetic sequence if the 3^{th} term is 50 and the 10^{th} term is 169.

Sum of Arithmetic Sequence

53. A big theater has 20 seats in the first row , 27 in the second, 34 in the third, and so on . If there are 30 rows in the theater, determine the number of seats in the theater.

54. The Drury lane theater has 25 seats in the first row and 30 rows in all. Each successive row contains once additional seat. How many seats are in the theater?

Determine if a sequence is geometric

55. Find the n^{th} term, a_n, of the following geometric sequence: 2, 1, ½, ¼ …

56. Find the n^{th} term, a_n, of the following geometric sequence: 10, -50, 250, -1250 …

Find the sum of a geometric sequence

57. Find the sum of $\sum_{k=1}^{15}\left(\frac{1}{3}\right)(2)^{k-1}$.

58. Find the sum of $\sum_{k=1}^{10}(5)\left(\frac{1}{2}\right)^{k-1}$.

Find the infinite sum of the geometric sequence

59. Find the infinite sum $\sum_{k=1}^{\infty}(3)\left(\frac{1}{4}\right)^{k-1}$.

60. Find the infinite sum $\sum_{k=1}^{\infty}(5)\left(\frac{1}{7}\right)^{k-1}$.

MAT 117 Final Exam Review Answers

Note: There is a reasonable assumption that most of these answers are not incorrect.

1. a. $[4, \infty)$ b. $[-5, \infty)$	31. a. $x = 3$ b. $x = 9$
2. a. $(-\infty,-5)\cup(-5,0)\cup(0,\infty)$ b. $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$	32. $t = 17.36$ years
3. a. $\left(\frac{7}{2},\infty\right)$ b. $\left(-\frac{4}{5},\infty\right)$	33. $t = 17.33$ years
4. $3 - 2x - h$	34. $t = 2271.9$ years
5. $-\frac{4}{x(x+h)}$	35. $t = 7402.8$ years
6. 0	36. $x = 2; y = -\frac{1}{5}$
7. –2	37. inconsistent
8. $g(x) = -(x-3)^2 - 1$	38. 12 lb of caramel corn and 8 lb of honey roasted nuts
9. $g(x) = -(x+2)^2 + 1$	39.senior citizens 215
10. $C(x) = .05x + 35$; \$44	40. a. $\frac{1}{16}, -\frac{2}{25}, \frac{3}{36}$ b. $\frac{2}{4}, \frac{4}{10}, \frac{8}{28}$
11. \$24	41. $a_2 = 26; \quad a_3 = 113$
12. $R(t) = -1.05t + 25.7$; 7.85	42. $a_2 = -4; \quad a_3 = -9$
13. 15,312.5 ft^2	43. $a_n = \frac{(-1)^{n+1}}{n^2}$

14. $x = 750$ units	44. $a_n = \dfrac{n+4}{3^n}$
15. a. $-x^2+4x-1$ b. x^2+4x+3	45. $\displaystyle\sum_{k=1}^{16}\frac{2k-1}{k+6}$
16. a. $x=2; x=3$ b. $x=-⅓$	46. $\displaystyle\sum_{k=1}^{11}(-1)^{k+1}\left(\frac{2}{3}\right)^k$
17. a. $y=-⅓$ b. $y=3$	47. 2415
18. $p = 2000$; As time increases the population stabilizes to 2000.	48. 871
19. $p = 3750$; The capacity of the pond is 3750 trout.	49. $a_{25} = 137$
20. $f(x)=(x-3)^2(x+1)^2$	50. $a_{20} = -128$
21. $f(x)=(x-1)^2(x+\frac{1}{2})^2$	51. $d = -4.5;\ a_1 = 21.5$
22. $f(x)=(x+6)(x-1)(x+\frac{1}{2})$; zeros –6, 1, –1/2	52. $d = 17;\ a_1 = 16$
23. $f(x)=(x+3)(x+i)(x-i)$; zeros –3, i, $-i$	53. 3645
24. $(f\circ g)(x)=9x^2-18x+5$ $(g\circ f)(x)=3x^2-12x-1$	54. 1185
25. $(f\circ g)(x)=x$; $(g\circ f)(x)=x$	55. $a_n = 2\left(\dfrac{1}{2}\right)^{n-1}$
26. $f^{-1}(x)=\dfrac{x+3}{x-2}$	56. $a_n = 10(-5)^{n-1}$
27. $g^{-1}(x)=\sqrt[3]{\dfrac{x+3}{4}}$	57. $32767/3 = 10922.\overline{3}$
28. $\log\left(\dfrac{x}{2(x+3)}\right)$	58. $5115/512 \approx 9.990234$
29. $\ln\left(\dfrac{x-1}{x+1}\right)$	59. 4
30. a. $x=\dfrac{\ln(6)}{\ln(3)}$ and $x=\dfrac{\ln(7)}{\ln(3)}$ b. $x=\dfrac{\ln(2)}{\ln(7)}$	60. 35/6