

MAT117 Review Problems for Exam 3

Evaluating an exponential function.

1. A stereo amplifier's power output P (in Watts) is related to its decibel voltage gain d by the formula:
 $P(d) = 25e^{0.1d}$. (approximate to one decimal place)
 - a. What is the power output when the decibel voltage gain is 4 decibels?
 - b. What is the power output when the decibel voltage gains is 12 decibels?
2. If a single plane of glass obliterates 3% of the light passing through it, the percent p of the light that passes through n successive panes is given approximately by the function $p(n) = 100(0.97)^n$. (approximate to one decimal place)
 - a. What percent of light will pass through 10 panes?
 - b. What percent of light will pass through 25 panes?

Determine the range of the exponential function

3. Determine the range of $f(x) = 5 - 2^{x+1}$.
4. Determine the range of $g(x) = e^{-3x} - 4$.

Change the exponential expression to a logarithmic expression

5. Write the equivalent logarithmic expression for $5^x = 125$.
6. Write the equivalent logarithmic expression for $e^y = 3x + 1$.

Change the logarithmic expression to an exponential expression

7. Write the equivalent exponential expression for $\ln(x) = 4$.
8. Write the equivalent exponential expression for $\log_2(x + 5) = 3$.

Domain of logarithms

9. Find the domain for the function $f(x) = \ln(3x - 18)$.
10. Find the domain for the function $f(x) = \log_3(2x + 5) - 4$.

Expand the logarithm

11. Expand $\ln\left(\frac{x^3 y^8}{z^5 w^2}\right)$ using the properties of logarithms.
12. Expand $\log\left(\frac{(x+3)^2}{5(x^2+1)^4}\right)$ using the properties of logarithms.

Compress to a single logarithm

13. Express as a single logarithm: $8 \log_2(x) - 5 \log_2(y) + 7 \log_2(z)$.
14. Express as a single logarithm and simplify: $\ln(x-3) - \ln(x^2-9) + 2 \ln(x)$.

Change of base property

15. Approximate $\log_8(12)$ using your calculator.
16. Find the value of $\log_3(243)$.

Exponential equations

Solve the equations.

17. $2^{2x} = 8^{x+1}$ 19. $5^{x+3} = 8$ 21. $3^{2x} - 15(3^x) + 56 = 0$
18. $4^{3x} = \frac{1}{16}$ 20. $7^{2x-3} = 9$ 22. $7^{2x} + 2(7^x) - 24 = 0$

Logarithmic equations

Solve the equations:

23. $8 \log_2(3x-1) - 7 = 17$ 26. $\log_2(2x) - \log_2(x-3) = 3$ 29. $\log_6(x^2 - x) = 1$
24. $2 \ln(5x-1) + 6 = 8$ 27. $\log(x) + \log(x-4) = \log(21)$ 30. $\ln(2x^2 - x) = 0$
25. $\ln(x+5) - \ln(17-3x) = 0$ 28. $\log_3(x) + \log_3(x-8) = 2$

Inverse functions

31. Find the inverse function of the following one-to-one functions:

- a. $f(x) = e^{x+5} - 4$ b. $g(x) = 2^{3x-2} + 5$

32. Find the inverse function of the following one-to-one functions:

- a. $f(x) = \ln(x-5) - 3$ b. $g(x) = \log(2x-3) + 1$

Compound Interest, $A = P(1 + \frac{r}{n})^{nt}$, $A = Pe^{rt}$

33. What amount will Mary have if she deposits \$5000 into an account for 10 years with 6% interest compounded:
- a. quarterly b. monthly c. continuously
34. How much money will Dylan need to invest in order to accumulate \$10,000 in 5 years with 8% interest compounded:
- a. quarterly b. monthly c. continuously
35. How long will it take for \$6000 to double if the money is deposited into an account that earns 4% interest compounded:
- a. quarterly b. monthly c. continuously

Exponential growth & decay, logistic model

36. A culture of bacteria obeys the law of uninhibited exponential growth.
- If 500 bacteria are present initially and there are 800 bacteria after 3 hours, how many will be present in 5 hours?
 - How long will it take for there to be 20,000 bacteria.
37. A piece of charcoal is found to contain 30% of the carbon-14 that it originally had. How old is the piece of charcoal? The half-life of carbon-14 is 5600 years.
38. The proportion of US households that own a DVD player can be represented by the logistic model function $P(t) = \frac{0.9}{1 + 6e^{-0.32t}}$. Let $t = 0$ represent the year 2000.
- When will the proportion of households reach 0.6 (60%)?
 - When will the proportion of households reach 0.8 (80%)?

Systems of equations in 2 variables

39. Solve the system of equations(or if it can't be solved say inconsistent), $\begin{cases} 7x - 5y = 15 \\ x + 5y = 1 \end{cases}$.
40. Solve the system of equations(or if it can't be solved say inconsistent), $\begin{cases} 8x - 3y = 5 \\ 16x - 6y = 7 \end{cases}$.
41. A restaurant manager wants to purchase 200 sets of dishes. One design with no pattern costs \$25 per set, while another with an American Eagle pattern costs \$45 per set. If she only has \$7400 to spend, how much of each design should be ordered?
42. A movie theater charges \$9 for adults and \$7 for senior citizens. On a day when 325 people paid admission, the total receipts were \$2495. How many people who paid were adults? How many were senior citizens?

Systems of equations in 3 variables

43. Solve the system of equations(or if it can't be solved say inconsistent), $\begin{cases} x - 2y + 3z = 7 \\ 2x + y + z = 4 \\ -3x + 2y - 2z = -10 \end{cases}$.
44. Solve the system of equations(or if it can't be solved say inconsistent), $\begin{cases} 3x - 2y + 2z = 6 \\ 7x - 3y + 2z = -1 \\ 2x - 3y + 4z = 0 \end{cases}$.

45. Patty wants to invest \$30,000 in corporate bonds. The bonds are rated AAA, AA, A, B, C, and those with lower ratings (more risk) pay a higher rate of interest. Currently the average yield is 5% on AAA bonds, 6% on A bonds and 10% on B bonds. Being conservative, Patty wants to have twice as much AAA bonds as in B bonds. Patty wants to guarantee an interest income of \$2000. Write the system only, do not solve.
46. Cartman's Snack Company wants to make a 100 pound mixture of corn chips, nuts and pretzels that will cost \$4 per pound. Corn chips cost \$2 per pound, nuts cost \$6 per pound, and pretzels cost \$3 per pound. The mixture has three times as many corn chips as pretzels (by weight). Write down the system only, do not solve.

Augmented matrix and row operations

47. Write the augmented matrix for the system of equations in problem 42.

48. Write the augmented matrix for the system of equations in problem 45.

49. Perform the following row operations $R_2 = -3r_1 + r_2$ on $R_3 = 2r_1 + r_3$ on $\left[\begin{array}{ccc|c} 4 & 8 & -4 & 4 \\ 3 & 6 & 5 & -13 \\ -2 & 1 & 12 & -17 \end{array} \right]$.

50. Perform the following row operations $R_2 = r_1 + r_2$ $R_3 = 3r_1 + r_3$ on $\left[\begin{array}{ccc|c} -3 & -5 & 36 & 10 \\ -1 & 0 & 7 & 5 \\ 1 & 1 & -10 & -4 \end{array} \right]$.

MAT 117 Exam 3 Review Answers

Note: There is a reasonable assumption that most of these answers are not incorrect.

1. a. 37.3 Watts b. 83.0 Watts	26. $x = 4$
2. a. 73.7 % b. 46.7%	27. $x = 7$
3. $(-\infty, 5)$	28. $x = 9$
4. $(-4, \infty)$	29. $x = 3, -2$
5. $x = \log_5 125$	30. $x = 1, -\frac{1}{2}$
6. $y = \ln(3x + 1)$	31. a. $f^{-1}(x) = \ln(x + 4) - 5$ b. $g^{-1}(x) = \frac{2 + \log_2(x - 5)}{3}$
7. $e^4 = x$	32. a. $f^{-1}(x) = e^{x+3} + 5$ b. $g^{-1}(x) = \frac{10^{x-1} + 3}{2}$
8. $2^3 = x + 5$	33. a. \$9070.01 b. \$9096.98 c. \$9110.59
9. $(6, \infty)$	34. a. \$6729.71 b. \$6712.10 c. \$6703.20
10. $(-\frac{5}{2}, \infty)$	35. a. 17.42 years b. 17.36 years c. 17.33 years
11. $3\ln(x) + 8\ln(y) - 5\ln(z) - 2\ln(w)$	36. a. 1094 b. 23.55 hours
12. $2\log(x+3) - \log(5) - 4\log(x^2 + 1)$	37. 9727 years
13. $\log_2 \left(\frac{x^8 z^7}{y^5} \right)$	38. a. 7.77 years b. 12.10 years

14. $\ln\left(\frac{x^2}{x+3}\right)$	39. $(2, -\frac{1}{5})$
15. approximately 1.195	40. inconsistent
16. 5	41. No pattern 80; American Eagle pattern 120
17. $x = -3$	42. Adults 110; Senior citizens 215
18. $x = -2/3$	43. $x = 2; y = -1; z = 1$
19. $x = \frac{\ln(8)}{\ln(5)} - 3$	44. inconsistent
20. $x = \frac{\ln(9)}{2\ln(7)} + \frac{3}{2}$	45. $\begin{cases} x + y + z = 30000 \\ x - 2z = 0 \\ .05x + .06y + .10z = 2000 \end{cases}$ $x = \text{AAA bonds}, y = \text{A bonds}, z = \text{B bonds}$
21. $x = \frac{\ln(7)}{\ln(3)}$ and $x = \frac{\ln(8)}{\ln(3)}$	46. $\begin{cases} x + y + z = 100 \\ x - 3z = 0 \\ 2x + 6y + 3z = 400 \end{cases}$ $x = \text{corn chip weight}, y = \text{nut weight}, z = \text{pretzel weight}$
22. $x = \frac{\ln(4)}{\ln(7)}$	47. $\left[\begin{array}{cc c} 1 & 1 & 325 \\ 9 & 7 & 2495 \end{array} \right]$
23. $x = 3$	48. $\left[\begin{array}{ccc c} 1 & 1 & 1 & 30000 \\ 1 & 0 & -2 & 0 \\ .05 & .06 & .10 & 2000 \end{array} \right]$
24. $x = \frac{e+1}{5}$	49. $\left[\begin{array}{ccc c} 1 & 2 & -1 & 1 \\ 0 & 0 & 8 & -16 \\ 0 & 5 & 10 & -15 \end{array} \right]$
25. $x = 3$	50. $\left[\begin{array}{ccc c} 1 & 1 & -10 & -4 \\ 0 & 1 & -3 & 1 \\ 0 & -2 & 6 & -2 \end{array} \right]$