

## MAT117 Review Problems for Exam 2

### Factoring Polynomials

1. Factor each polynomial;

a.  $3x^2 - 10x + 8$       b.  $2x^2 + 5x + 3$

2. Factor each polynomial;

a.  $6x^3 + 9x^2 + 4x + 6$       b.  $3x^3 + 6x^2 - x - 2$       c.  $x^3 - x^2 - 16x + 16$

### Quadratic function Vertex/Standard form

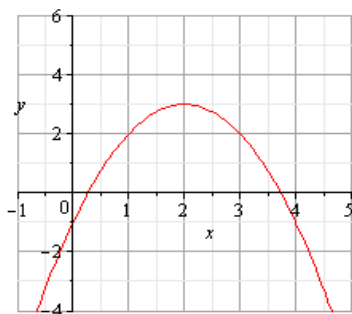
3. Write the following functions in standard form,  $f(x) = a(x - h)^2 + k$ .

a.  $f(x) = 3x^2 - 12x + 15$ .      b.  $f(x) = -5x^2 + 30x - 25$ .

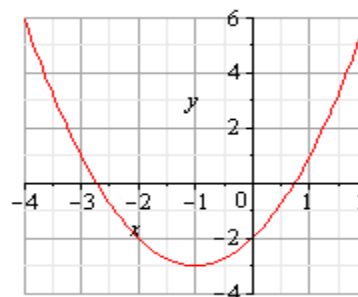
4. Graph the following functions to determine their range;

a.  $f(x) = -2x^2 - 6x + 15$ .      b.  $f(x) = 3x^2 + 6x - 1$ .

5. Given the graphs of the quadratic functions, determine their definition (formula) from their graph.



a. vertex = (2,3)



b. vertex = (-1,-3)

### Application of Quadratic Function

6. The John Deere company has found that the revenue, in dollars, from sales of heavy-duty tractors is a function of the unit price  $p$ , in dollars, that is charges. If the revenue  $R$  is  $R(p) = -0.5p^2 + 1900p$ .

- What unit price should be charged to maximize revenue?
- What is the maximum revenue?

7. Consider the quadratic model  $h(t) = -16t^2 + 64t + 10$  for the height, in feet, of an object  $t$  seconds after the object has been projected straight up into the air.

- What is the time required for the object to reach the maximum height?
- What is the maximum height?

## Polynomials and their degree;

8. Given  $f(x) = -3(x^2 + 1)(x - 2)^3$ .

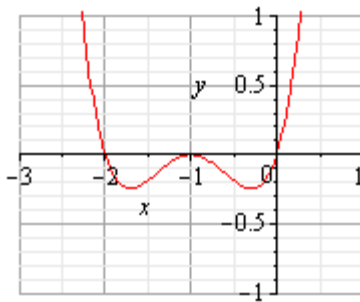
- State whether  $f(x)$  is a polynomial function or not. If it is a polynomial, give the degree. If it is not a polynomial, tell why not.
- Determine the behavior of the graph of  $f(x)$  near each root(zero).
- Determine whether the graph of  $f(x)$  touches or crosses the  $x$ -axis, at each root(zero).

9. Given  $f(x) = 7(x + 3)^2(x - 4)$ .

- State whether  $f(x)$  is a polynomial function or not. If it is a polynomial, give the degree. If it is not a polynomial, tell why not.
- Determine the behavior of the graph of  $f(x)$  near each root(zero).
- Determine whether the graph of  $f(x)$  touches or crosses the  $x$ -axis, at each root(zero).

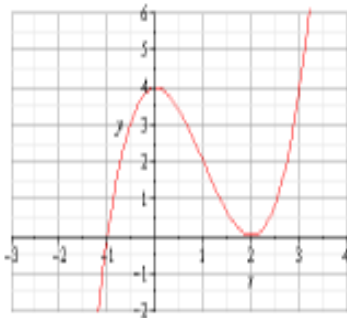
10. Which of the following polynomial functions might have the graph shown below?

a.  $f(x) = x(x + 1)^2(x + 2)$       b.  $g(x) = x(x - 1)^2(x + 2)$       c.  $h(x) = x(x + 1)^2(x - 2)$



11. Which of the following polynomial functions might have the graph shown below?

a.  $f(x) = (x + 1)^2(x + 2)$       b.  $g(x) = (x + 1)(x + 2)^2$       c.  $h(x) = (x + 1)(x - 2)^2$



## Real zeros and multiplicities of polynomial

12. For the function  $g(x) = -2(x + 7)^3(x - 1)^2$ , list each real zero and its multiplicity. Determine whether the graph crosses or touches the  $x$ -axis at each  $x$ -intercept.

13. For the function  $g(x) = 3(x - 5)(x + 1)^2$ , list each real zero and its multiplicity. Determine whether the graph crosses or touches the  $x$ -axis at each  $x$ -intercept.

14. Form a degree 3 polynomial,  $p(x)$  with leading coefficient of 1 with zeros:  $-2$  of multiplicity 2 and 3 of multiplicity 1.

15. Form a degree 3 polynomial,  $p(x)$  with leading coefficient of 1 with zeros: 3 of multiplicity 1 and  $-5$  of multiplicity 2.

### Rational function – domain

16. Find the domain of the following rational functions:

a.  $f(x) = \frac{x-1}{3x^2-11x-4}$ .      b.  $g(x) = \frac{2x+7}{4x^2+3x-10}$ .

### Vertical asymptotes & Horizontal asymptotes

17. Find the equation of the vertical asymptotes of the following functions:

a.  $f(x) = \frac{1-x^2}{x^2-5x+6}$ .      b.  $g(x) = \frac{x^2-4}{3x^2-5x-2}$ .

18. Find the equation of the horizontal asymptotes of the following functions:

a.  $f(x) = \frac{1-x^2}{x^2-5x+6}$ .      b.  $g(x) = \frac{6x^2+x+12}{3x^2-5x-2}$ .

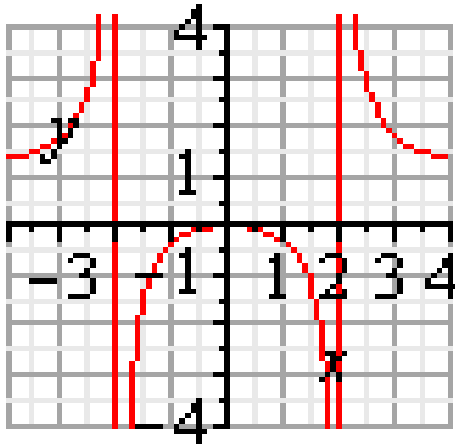
19. The population  $P$  for an insect  $t$  months after being transplanted is  $P(t) = \frac{50(1+0.5t)}{(2+0.01t)}$ . Determine the horizontal asymptote of  $P(t)$ . Describe this in the context of the problem.

20. A company that produces scooters has an average cost given by the function  $\bar{C}(x) = \frac{50x+30001}{2x}$ .

What is the horizontal asymptote for the function  $\bar{C}(x)$ ? Describe this in the context of the problem.

### Graphs of rational functions

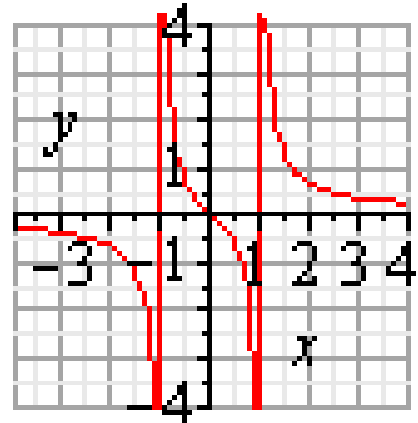
21. Find a rational function (formula) that might have the given graphs.



a.

22. Given  $Q(x) = \frac{2x+8}{3x-15}$ , find the  $x$ -intercept and  $y$ -intercept.

23. Given  $Q(x) = \frac{x^2-1}{x^2-16}$ , find the  $x$ -intercept and  $y$ -intercept



b.

**Rational zero test**

24. List the **possible** rational zeros for the polynomial  $p(x) = 2x^4 + 4x^3 - 3x^2 + 6$ .

25. List the **possible** rational zeros for the polynomial  $p(x) = 5x^4 + 4x^3 - 3x^2 + 20$ .

26. Suppose  $x = -6$  is a zero of  $p(x) = 2x^3 + 11x^2 - 7x - 6$  Find the rest of the real zeros to factor  $f(x)$  completely.

27. Suppose  $x = 1$  is a zero of  $p(x) = x^3 + 8x^2 + 11x - 20$  Find the rest of the real zeros to factor  $f(x)$  completely.

**Find all the zeros / complex zeros**

28. Write a polynomial  $f(x)$  whose coefficients are real numbers, degree 3, zeros at  $-1$  and  $2i$ .

29. Write a polynomial  $f(x)$  whose coefficients are real numbers, degree 3, zeros at  $3$  and  $1-2i$ .

30. Find **all** the zeros of the function,  $f(x) = x^3 - 4x^2 + x - 4$  and write the polynomial as a product of linear factors.

31. Find **all** the zeros of the function,  $f(x) = x^3 - 3x^2 + 12x - 10$  and write the polynomial as a product of linear factors.

**Composite functions**

32. Given  $f(x) = \frac{1-2x}{3x}$  and  $g(x) = 3x^2 - 9x$ , find the following

a.  $(g \circ f)(-1)$ .      b.  $(f \circ g)(2)$ .

33. Let  $f(x) = 3x^2 - 2x$ , and  $g(x) = 1 - 4x$ , find the following:

a.  $(f \circ g)(x)$ .      b.  $(g \circ f)(x)$ .

34. Let  $f(x) = x^2 + 1$ , and  $g(x) = \sqrt{x-1}$ , find the following:

a.  $(f \circ g)(x)$ .      b.  $(g \circ f)(x)$ .

35. Find the functions  $f(x)$  and  $g(x)$ , so that  $H(x) = (f \circ g)(x) = \sqrt{4-5x}$

36. Find the functions  $f(x)$  and  $g(x)$ , so that  $H(x) = (f \circ g)(x) = |2x^3 - 1|$

### Inverse functions

37. Given  $y = f(x)$  is a one-to-one function, suppose  $f(2) = -5$ . What is  $f^{-1}(-5)$ ?

38. Given  $y = f(x)$  is a one-to-one function, suppose  $f(a) = b$ . What is  $f^{-1}(b)$ ?

39. Find the inverse function,  $f^{-1}(x)$ , given the one-to-one function  $f(x) = \frac{x-5}{2x+3}$ .

40. Find the inverse function,  $g^{-1}(x)$ , given the one-to-one function  $g(x) = 2x^3 - 1$ .

## MAT 117 Test 2 Review Answers

**Note: There is a reasonable assumption that most of these answers are not incorrect.**

1. a. $(3x-4)(x-2)$ b. $(x+1)(2x+3)$	21. a. $f(x) = \frac{x^2}{(x+2)(x-2)}$ b. $f(x) = \frac{x}{(x-1)(x+1)}$
2. a. $(3x^2+2)(2x+3)$ b. $(3x^2-1)(x+2)$ c. $(x-1)(x-4)(x+4)$	22. x-int $(-4, 0)$ , y-int $(0, -\frac{8}{15})$
3. a. $f(x) = 3(x-2)^2 + 3$ b. $f(x) = -5(x-3)^2 + 20$	23. x-int $(-1, 0)$ and $(1, 0)$ , y-int $(0, \frac{1}{16})$
4. a. $(-\infty, 19.5]$ b. $[-4, \infty)$	24. $\pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm 3, \pm 6$
5. a. $f(x) = -x^2 + 4x - 1$ b. $f(x) = x^2 + 2x - 2$	25. $\pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{4}{5}, \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$
6. a. 1900    b. \$1,805,000	26. $f(x) = (x+6)(x-1)(2x+1)$
7. a. 2 seconds    b. 74 feet	27. $f(x) = (x-1)(x+4)(x+5)$
8. a. polynomial degree 5 b. $f(x) \approx -15(x-2)^3$ near $x = 2$ . c. crosses the x-axis at $x = 2$ .	28. $f(x) = x^3 + x^2 + 4x + 4$
9. a. polynomial degree 3 b. $f(x) \approx -49(x+2)^2$ near $x = -3$ ; $f(x) \approx 343(x-4)$ near $x = 4$ . c. touches the x-axis at $x = -3$ . crosses the x-axis at $x = 4$ .	29. $f(x) = x^3 - 5x^2 + 11x - 15$

10. a	30. $4, i, -i$ $f(x) = (x-4)(x-i)(x+i)$
11. c	31. $1, 1+3i, 1-3i$ $f(x) = (x-1)(x-1+3i)(x-1-3i)$
12. $-7$ of multiplicity 3 and the graph crosses, $1$ of multiplicity 2 and the graph touches	32. a. $12$ b. $-\frac{13}{18}$
13. $5$ of multiplicity 1 and the graph crosses, $-1$ of multiplicity 2 and the graph touches	33. a. $48x^2 - 16x + 1$ b. $1 + 8x - 12x^2$
14. $p(x) = (x+2)^2(x-3) = x^3 + x^2 - 8x - 12$	34. a. $x$ b. $x$
15. $p(x) = (x-3)(x+5)^2 = x^3 + 7x^2 - 5x - 75$	35. $f(x) = \sqrt{x}$ ; $g(x) = 4 - 5x$
16. a. $(-\infty, -\frac{1}{3}) \cup (-\frac{1}{3}, 4) \cup (4, \infty)$ b. $(-\infty, -2) \cup (-2, \frac{5}{4}) \cup (\frac{5}{4}, \infty)$	36. $f(x) =  x $ ; $g(x) = 2x^3 - 1$
17. a. $x = 2$ and $x = 3$ b. only $x = -\frac{1}{3}$	37. $2$
18. a. $y = -1$ b. $y = 2$	38. $a$
19. H.A. $P = 2500$ , As time passes the population stabilizes to 2500.	39. $f^{-1}(x) = \frac{-5-3x}{2x-1}$
20. H.A. $\bar{C} = \$25$ , As the number of units increases the average cost stabilizes to \$25	40. $g^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}$