

ERRATA FOR “A SHARP CONDITION FOR SCATTERING OF THE RADIAL 3D CUBIC NONLINEAR SCHRÖDINGER EQUATION”

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ABSTRACT. We correct a mistake with the Strichartz estimates in our CMP paper. We thank Cristi Guevara and Fernando Carreon for pointing out this mistake.

The pair $(\frac{5}{3}, 10)$ is not an L^2 admissible Strichartz pair. Thus, our article needs straightforward modification in the following places.

0.1. Modifications to the proof of Prop. 2.1. *Starting with the line “Applying the Strichartz ...” replace by the following:* Applying the Strichartz estimates, we obtain

$$\|D^{1/2}\Phi_{u_0}(v)\|_{S(L^2)} \leq c\|u_0\|_{\dot{H}^{1/2}} + c\|D^{1/2}(|v|^2v)\|_{L_t^{10/7}L_x^{10/7}}$$

and

$$\|\Phi_{u_0}(v)\|_{S(\dot{H}^{1/2})} \leq \|e^{it\Delta}u_0\|_{S(\dot{H}^{1/2})} + c\|D^{1/2}(|v|^2v)\|_{L_t^{10/7}L_x^{10/7}}.$$

Applying the fractional Leibnitz [18] and Hölder inequalities,

$$\|D^{1/2}(|v|^2v)\|_{L_t^{10/7}L_x^{10/7}} \leq \|v\|_{L_t^5L_x^5}^2 \|D^{1/2}v\|_{L_t^{10/3}L_x^{10/3}} \leq \|v\|_{S(\dot{H}^{1/2})}^2 \|D^{1/2}v\|_{S(L^2)}.$$

The remainder of the proof is the same.

0.2. Modifications to the proof of Prop. 2.2. *At the beginning of the proof, insert the following lines:*

Let $M = \|u\|_{L_t^5L_x^5} < \infty$. We claim that $\|\langle \nabla \rangle u\|_{S(L^2)} \lesssim BM^5$, where $\langle \nabla \rangle = (I - \Delta)^{1/2}$. Decompose $[0, +\infty) = \cup_{j=1}^{\tilde{M}} I_j$, where $\tilde{M} \sim M^5$, and for each j , $\|u\|_{L_{I_j}^5L_x^5} \leq \delta$. Applying $\langle \nabla \rangle$ to the integral equation on I_j and then applying the Strichartz estimates, we obtain

$$\begin{aligned} \|\langle \nabla \rangle u\|_{S(L^2; I_j)} &\leq B + \|u^2 \langle \nabla \rangle u\|_{L_{I_j}^{10/7}L_x^{10/7}} \\ &\leq B + \|u\|_{L_{I_j}^5L_x^5}^2 \|\langle \nabla \rangle u\|_{L_{I_j}^{10/3}L_x^{10/3}} \\ &\leq B + \delta^2 \|\langle \nabla \rangle u\|_{S(L^2; I_j)} \end{aligned}$$

From this we conclude

$$\|\langle \nabla \rangle u\|_{S(L^2; I_j)} \leq 2B$$

and by summing over the \tilde{M} intervals, we conclude the proof of the claim.

After the definition of ϕ_+ , replace the rest of the proof with the following: We first show that $\phi_+ \in H^1$. Indeed, by the Strichartz estimates and the definition of ϕ_+ ,

$$\begin{aligned} \|\phi_+\|_{H^1} &\leq \|u_0\|_{H^1} + \|u^2 \langle \nabla \rangle u\|_{L_{[0,+\infty)}^{10/7} L_x^{10/7}} \\ &\leq \|u_0\|_{H^1} + \|u\|_{L_{[0,+\infty)}^5 L_x^5}^2 \|\langle \nabla \rangle u\|_{L_{[0,+\infty)}^{10/3} L_x^{10/3}} \\ &\lesssim B + BM^7. \end{aligned}$$

Applying the Strichartz estimates to (2.3),

$$\begin{aligned} \|u(t) - e^{it\Delta} \phi_+\|_{H^1} &\leq c \|u^2 \langle \nabla \rangle u\|_{L_{[t,+\infty)}^{10/7} L_x^{10/7}} \\ &\leq \|u\|_{L_{[t,+\infty)}^5 L_x^5}^2 \|\langle \nabla \rangle u\|_{L_{[t,+\infty)}^{10/3} L_x^{10/3}} \end{aligned}$$

and the right side $\rightarrow 0$ as $t \rightarrow +\infty$.

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