Math 274 Practice Problems for Exam 1

The Exam 1 will cover:
- Chapter 2 (sections 2.1, 2.2, 2.3, 2.5, 2.6) and
- Chapter 3 (sections 3.1-3.7).

1. Solve the Initial Value Problems:

   (a) $yy' = x(y^2+1)$, $y(0) = 0$ (answer: $y = \sqrt{e^{x^2}-1}$)

   (b) $xy' - y = x$, $y(1) = 1$ (answer: $y = x\ln(x) + x$)

   (c) $(x+y)y' = 1 - x - y$, $y(1) = 1$ (hint: use substitution $v = x + y$) (answer: $y = \sqrt{2x^3 - x^2}$)

   (d) $xyy' = x^2 + 2y^2$, $y(1) = 0$ (answer: $y^2 = x^4 - x^2$)

   (e) $(1+x)y' = 4y$, $y(0) = 3$ (answer: $y = 3(1+x)^4$)

   (f) $xy' = 3y + x^4\cos(x)$, $y(\pi/2) = 0$ (answer: $y = (\sin x - 1)x^3$)

   (g) Also review problems on p.131.

2. Which of the following equations are exact? Find the general solution for exact.

   (a) $(1/x - 1)dx + 1/ydy = 0$
   (b) $(ye^{xy} - y)dx + (xe^{xy} - y)dy = 0$
   (c) $ye^y dx + (1+y)xe^y dy = 0$

   (answers: (a) exact, $\ln(xy) - x = c$;
   (b) not exact, can you use integrating factor?
   (c) exact, $xye^y = c$)

3. Are the functions $y_1 = 1$, $y_2 = e^{2x}$ and $y_3 = xe^{2x}$ linearly independent?

   (answer: $W = 4e^{4x} \neq 0$ - linearly independent)

Also review Section 3.2 problems # 1-6 and # 17-19.

4. (a) Find the general solution of $y'' - y' - 2y = 0$.

   (b) Solve: $y'' - y' - 2y = 8e^{3x}$

   (answer: $y = y_h + y_p = c_1 e^{2x} + c_2 e^{-x} + 2e^{3x}$)

5. Verify that $y_1(t) = 1 + t$ and $y_2(t) = e^t$ satisfy the corresponding homogeneous equation, then find a particular solution of $ty'' - (1 + t)y' + y = t^2 e^{2t}$, $t > 0$.

6. For section 2.3 review problems # 7-18.

7. For section 2.5 review problems #1-6, 8-13, 15-17, 20-23, 25-27.

8. Solve $y''' + 9y' = 0$. Write down an appropriate guess for y_p for $y''' + 9y' = e^x + 4\cos 3x$.

   (answer: $y_h = c_1 + c_2 \cos 3x + c_2 \sin 3x$;
   $y_p = Ae^x + (B\cos 3x + C\sin 3x)x$)

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1. (a) Find the general solution of $y' = -\frac{3x^2y+x}{x^3+y^3+1}$. You may leave your answer in implicit form.

   (b) Find the particular solution to the equation in part (a) that satisfies $y(0) = 1$. You may leave this answer in implicit form also.

2. (a) Find the general solution of
$$\frac{dy}{dx} = \frac{x^5 + 2y}{x}.$$
Your answer should be in explicit form, that is, in the form $y = f(x)$.

   (b) Check your answer to part (a) by plugging your formula for y into the equation in part (a) and seeing if the equation is satisfied.

3. (a) Find the general solution to the equation $\frac{dy}{dt} - y^5 = 0$, with $y(0) \neq 0$. Your answer should be in explicit form, that is, in the form $y = f(t)$.

   (b) Find the particular solution to the equation in part (a) that satisfies $y(0) = y_0$, where $y_0 \neq 0$ is a given number.

   (c) For what values of t is your solution in part (b) defined?

4. Solve the equation $xy' = y - xe^{y/x}$. Your answer should be in explicit form, that is, in the form $y = f(x)$.

5. Solve
$$y'' - 4y' + 5y = 0, \qquad y(0) = 3, \qquad y'(0) = 4.$$

6. Consider the equation
$$(\star) \qquad t^2y'' - 5ty' + 8y = 0.$$
   (a) Check that $y_1 = t^2$ is a solution of $(\star)$.

   (b) Find the general solution of $(\star)$.

7. Find the general solution of
$$y'' + 2y' - 3y = -9t^2 + 8e^t.$$

8. For the equation
$$y'' - 4y' + 4y = t^3e^{2t} + t^2e^{3t} + t\sin 4t,$$
write down the general form of the particular solution y_p that you would use in the method of undetermined coefficients. Just write down the form; do not attempt to solve for the coefficients.

9. Assume the following fact: the functions $y_1 = t^2$ and $y_2 = t^{-1}$ are solutions of
$$(\star) \qquad y'' - \frac{2}{t^2}y = 0, \qquad t > 0.$$

(a) Prove that y_1 and y_2 are linearly independent, and write down the general form of the solution of $(\star)$.

(b) Find the general solution of

$$y'' - \frac{2}{t^2}y = 3t^3, \qquad t > 0.$$

10. Suppose that a certain population obeys the logistic equation patterns:

(a) $\dfrac{dy}{dt} = -\alpha y(1 - \dfrac{y}{K})(1 - \dfrac{y}{T})$ with $T > K$.
(a) Determine equilibrium points of this equation and provide the stability analysis. (b) What happens to the population if $y_0 > T$? if $K < y_0 < T$?

(b) $\dfrac{dy}{dt} = y^2(1 - y)$.
(a) Do the stability analysis; (b) Solve the equation; (c) Find what happens to the population in the long run if $y_0 = .5$.