Notes – Random Variables – Math 211

In Chapter 12 we calculated probabilities using numeric methods. This method works fine if the sample space S and the event spaces E can be easily manipulated from a combinatorics standpoint. In Chapter 13 we begin to treat probabilities as functions. We will then be able to use mathematical models and even use calculus to help answer probability questions. The first step is to introduce *random variables*.

A random variable is usually written as a capital X (or Y, etc.). Technically, it is a function that assigns numeric values to the outcomes of a sample space. In most cases the values assigned by X are very logical.

For example, if an experiment consisted of rolling two dice, we could define random variable X as the sum of the two faces. If the dice came up 3 and 4, we'd say that $X = 7$. This is very logical; we have 'assigned' the value of 7 to this outcome. Note that in this scenario, X can equal any integer between 2 and 12, inclusive.

If we were flipping three coins we could define random variable X to equal the number of heads that result. The possible values for X would be 0, 1 2 or 3.

Next, we determine the probabilities for these possible assignments. The symbolic sentence would look like this:

$$p(X = x)$$

In English this is read as "the probability that random variable X is equal to x, where x is the value associated with some experiment". In the rolling of two dice, for example, the probabilities would be:

$$p(X=2)=\tfrac{1}{36},\quad p(X=3)=\tfrac{2}{36},\quad p(X=4)=\tfrac{3}{36},\quad p(X=5)=\tfrac{4}{36},\quad p(X=6)=\tfrac{5}{36},$$
$$p(X=7)=\tfrac{6}{36},\quad p(X=8)=\tfrac{5}{36},\quad p(X=9)=\tfrac{4}{36},\quad p(X=10)=\tfrac{3}{36},\quad p(X=11)=\tfrac{2}{36},\quad p(X=12)=\tfrac{1}{36}$$

Please note that these probabilities are the same as before. Random variables do not 'change' the way we calculate probabilities. They simply allow us to be more mathematically accurate with our notation. It may take some time to get used to this notation, and in time you will see its advantages. Can you establish the probabilities for the flipping of three coins cited in the other example, using random variable notation?

Most importantly, we now have a way to create functions whose outputs are probabilities. In the rolling of two dice situation above, we could say that $f(2)=\tfrac{1}{36}$ for example. In general, these functions whose outputs are probabilities are called *probability mass functions* (pmf), and they meet the following requirements:

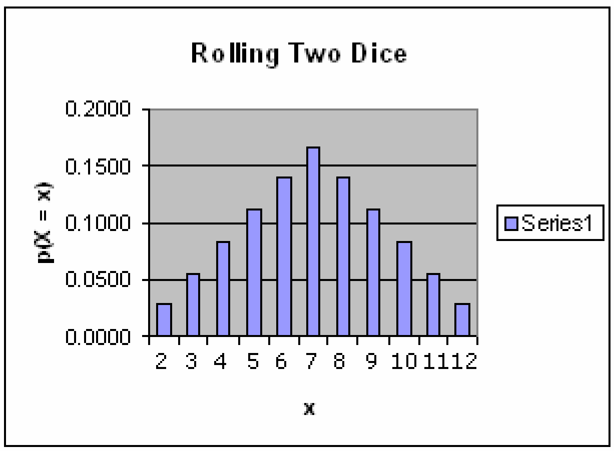
Given $f(x)$ is a pmf:

1. The output of $f(x)$ is the probability of x occurring, i.e. $f(x) = p(X = x)$.
2. $0 \le f(x) \le 1$ for all x.
3. The sum of all values $f(x)$ = 1.

The pmf $f(x)$ for the rolling of two dice has this somewhat awkward definition:

$$f(x) = p(X = x) = \begin{cases} \frac{1}{36} & x = 2 \text{ or } x = 12 \\ \frac{2}{36} & x = 3 \text{ or } x = 11 \\ \frac{3}{36} & x = 4 \text{ or } x = 10 \\ \frac{4}{36} & x = 5 \text{ or } x = 9 \\ \frac{5}{36} & x = 6 \text{ or } x = 8 \\ \frac{6}{36} & x = 7 \\ 0 & \text{otherwise} \end{cases}$$

BUT… we can now treat the probabilities as a function, which in turn we can graph as a histogram:



This gives us a nice visual representation of how the probabilities for this particular problem 'look' graphically.

Practice problems:

1. Roll two dice and let random variable X equal the average of the two numbers showing face up. Following the same procedure in the above example, (a) write out the probabilities using the random variable notation, (b) create a function $f(x)$ and (c), sketch a probability histogram.

2. A bag contains a \$1 bill, a \$5 bill, and a \$10 bill. You reach in and grab a bill, then you put it back in the bag. You reach in again and randomly grab a bill. (i.e. 'with replacement'). Let X be the total dollar amount you grabbed. Do the same steps as in the previous problems and examples.