Integration Practice - Mat 210

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I. Basic antiderivatives. Using the properties from your text, determine the general antiderivative below.

a) $\int 2\,dx$

b) $\int \left(x^2-1\right)dx$

c) $\int \left(4x^2+2x+7\right)dx$

d) $\int \left(\sqrt{x}+\frac{1}{x}\right)dx$

e) $\int \left(\frac{2}{x}+\frac{2}{x^3}\right)dx$

f) $\int 4.782x^{1.545}+3.713x^{0.762}\;dx$

g) $\int 35(2)^x\;dx$

h) $\int 6.782(3.335)^{2x}\;dx$ (Hint: rewrite the base)

i) $\int 26.86e^{0.65x}\;dx$ (Same hint as above)

Answers:

a) $F(x) = 2x + C$

b) $F(x) = \frac{1}{3}x^3 - x + C$

c) $F(x) = \frac{4}{3}x^3 + x^2 + 7x + C$

d) $F(x) = \frac{2}{3}x^{3/2} + \ln x + C$

e) $F(x) = 2\ln x - \frac{1}{x^2} + C$

f) $F(x) = 1.878x^{2.545} + 2.107x^{1.762} + C$

g) $F(x) = 50.49(2)^x + C$

h) $F(x) = 2.815(11.122)^x + C$

i) $F(x) = 41.323(1.916)^x + C$

II. Initial conditions. For each function below, find the particular antiderivative that satisfies the given initial condition.

a) $\int 4x+7\;dx;\quad F(2)=3$

b) $\int \left(\frac{1}{x}+\frac{1}{x^2}\right)dx;\quad F(1)=5$

c) The velocity of a particle is governed by the equation $v(t) = 2t^2 + t$, where t is in minutes and $v(t)$ is in feet/minute. Find the distance function $D(t)$ for this particle given that after 5 minutes, the particle had traveled 30 feet.

Answers:

a) $F(x) = 2x^2 + 7x - 18$

b) $F(x) = \ln x - \frac{1}{x} + 6$

c) $D(t) = \frac{2}{3}t^3 + \frac{1}{2}t^2 - 65.833$

III. Definite Integrals. Evaluate the following definite integrals.

a) $\int_0^2 (5x-6)\,dx$

b) $\int_{-1}^5 \left(4x^2+x-1\right)dx$

c) $\int_4^9 \sqrt{x}\,dx$

d) $\int_1^4 \left(\frac{3}{x}+x\right)dx$

e) $\int_{\ln 2.4}^{\ln 5} e^x\,dx$

f) $\int_{0.7}^{1.3} 1.488(2.716)^x \, dx$

g) The rate of change in a town's population is given by the function $f(x) = 169.147(1.07)^x$, where x is years since 1980 and $f(x)$ is people/year. What was the overall change in the town's population between 1980 and 1990?

Answers to definite integrals:

a) -2

b) 174

c) $\frac{38}{3}$

d) 7.5 + 3 ln 4

e) 2.6

f) 2.461

g) 2,417 people.

IV. Area Between Two Curves. Set up the integrals and determine the area between the two given curves between the specified bounds. If no bounds are given you will need to determine them (points of intersection).

a) $f(x) = x^2$, $g(x) = x - 2$, $-2 \leq x \leq 3$

b) $f(x) = 2e^x$, $g(x) = (1.05)^x$, $2 \leq x \leq 4$

c) Area enclosed within the curves $f(x) = x^2 - 3$ and $g(x) = x - x^2$

Answers:

a) $\frac{115}{6}$

b) 92.16

c) The intersection points are $x = -1$ and $x = 1.5$. The integral would be $\int_{-1}^{1.5} \left(\left(x - x^2\right) - \left(x^2 - 3\right)\right) dx = 5.208$