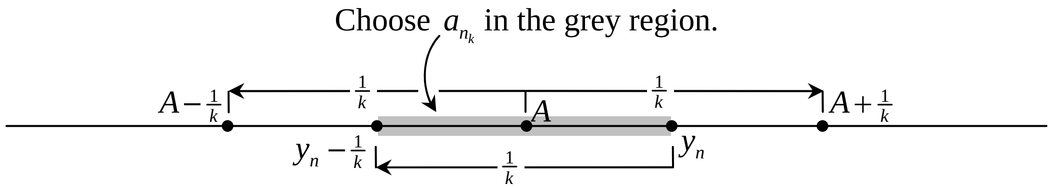
Guidance for Project 1.7

Following these suggestions, you can construct a subsequence $\left\{a_{n_k}\right\}_{k=1}^{\infty}$ that converges to A as shown in the diagram.



To start off with $k=1$, explain why you can pick $y_n \in [A, A+1)$ for some n. Then explain why you can choose $n_1 \geq n$ with $a_{n_1} \in (y_n - 1, y_n]$. Now repeat this process recursively to choose the remaining terms: for $k \in \mathbb{N}$ with $k > 1$, explain why can update the value of n with $n > n_{k-1}$ (i.e., bigger than the index of the previous term you picked) and $y_n \in \left[A, A+\frac{1}{k}\right)$. Then explain why you can choose $n_k \geq n$ with $a_{n_k} \in \left(y_n - \frac{1}{k}, y_n\right]$. Argue that this means $n_k > n_{k-1}$ and $a_{n_k} \in \left(A-\frac{1}{k}, A+\frac{1}{k}\right)$. To show that $\left\{a_{n_k}\right\}_{k=1}^{\infty}$ converges to A, use a cut-off $K > \dfrac{1}{\varepsilon}$.