Emerging Scholars
University of Texas

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# WORKSHEET 17

## Practice Exam

1. Find parametric equations for the line through $P(1, 7, -1)$ parallel to the $xy$ plane and perpendicular to
$$\frac{3x-1}{2} = 2 - 2y = \frac{1-z}{2}.$$

2. Let $\overline{\mathbf{r}}(t) = (1 + \sin t)\overline{\mathbf{i}} - (\cos t)\overline{\mathbf{j}}$.
   a) Sketch the curve.
   b) Find the velocity vector $\overline{\mathbf{r}}'$ at $t = \pi$ and sketch it with its tail at $\overline{\mathbf{r}}(\pi)$.
   c) Find the acceleration vector $\overline{\mathbf{r}}''$ at $t = \pi$ and sketch it with its tail at $\overline{\mathbf{r}}(\pi)$.
   d) Compute $\overline{\mathbf{r}}'(\pi) \cdot \overline{\mathbf{r}}''(\pi)$
   e) What is $\frac{d}{dt}||r'(t)||$ at $t = \pi$?

3. Do the four points
$$P(1, 1, 1) \qquad Q(2, 1, -1) \qquad R(3, -1, 1) \qquad S(0, 0, 0)$$
all lie in the same plane? Why or why not?

4. Rewrite Problem 3 in terms of vectors.

5. Find the plane through $P(-1, -1, -1)$ perpedicular to
$$(3\overline{\mathbf{i}} - \overline{\mathbf{k}}) \cdot ((x-2)\overline{\mathbf{i}}, y\overline{\mathbf{j}}, z\overline{\mathbf{k}}) = 0 \qquad \text{and} \qquad 2x - 3y + 7z + 7 = 0.$$

6. Find the area of the parallelogram with vertices
$$P(1, 2, 1) \qquad Q(2, 3, -3) \qquad R(0, 0, 5) \qquad S(1, 1, 1).$$

7. Let $\overline{\mathbf{u}} = \overline{\mathbf{i}} - 2\overline{\mathbf{j}} + 2\overline{\mathbf{k}}$ and $\overline{\mathbf{v}} = \overline{\mathbf{i}} + \overline{\mathbf{j}} - \overline{\mathbf{k}}$
   a) Find the angle between the vectors.
   b) Compute $\mathbf{proj}_{\overline{\mathbf{v}}}\overline{\mathbf{u}}$.
   c) Compute $\mathrm{comp}_{\overline{\mathbf{u}}}\overline{\mathbf{v}}$.

8. Find the equation of the intersection of the two planes given in Problem 5.

9. a) Are the line $\overline{\mathbf{r}} = (1 + t)\overline{\mathbf{i}} + \overline{\mathbf{j}} - t\overline{\mathbf{k}}$ and the vector $\overline{\mathbf{i}} - \overline{\mathbf{j}}$ parallel? Are they perpendicular?
   b) Find the projection of the vector in part a) onto the line in part a).

10. Find the unit normal to the plane $z = x - 2y$.

11. Let $\overline{\mathbf{u}}$ be a vector in $\mathbb{R}^2$ that makes a $45^\circ$ angle with $-\overline{\mathbf{i}} + \overline{\mathbf{j}}$ and has length 2. What is $\overline{\mathbf{u}}$?

12. Do the points
$$P(1,2,1) \qquad Q(0,-1,2) \qquad R(-3,-3,2)$$
all lie on a common line? Why or why not?

13. Let $\overline{\mathbf{r}} = (1+t^2)\overline{\mathbf{i}} - (1-t)\overline{\mathbf{j}}$.
    a) Sketch the curve.
    b) Find the velocity vector $\overline{\mathbf{r}}'$ at $t = 0$ and sketch it with its tail at $\overline{\mathbf{r}}(0)$.
    c) Find the acceleration vector $\overline{\mathbf{r}}''$ at $t = 0$ and sketch it with its tail at $\overline{\mathbf{r}}(0)$.
    d) Compute $\overline{\mathbf{r}}'(0) \cdot \overline{\mathbf{r}}''(0)$
    e) What is $\frac{d}{dt}||r'(t)||$ at $t = 0$?

14. Find the cross product
$$(\overline{\mathbf{j}} + 2\overline{\mathbf{k}}) \times (-i + 2\overline{\mathbf{k}}).$$

15. Find the plane parallel to
$$\frac{x-2}{3} = 2 - 2y = \frac{z}{7}$$
containing the points $(1,2,1)$ and $(0,0,0)$

16. Find symmetric and scalar parametric equations for the line parallel to $x - y - z = 1$ and perpendicular to $\overline{\mathbf{v}} = i + 2\overline{\mathbf{j}} - \overline{\mathbf{k}}$ through the point $P = (2,-2,1)$.

17. Find the equation of the plane containing the line
$$1 - 3x = \frac{1}{2}y = \frac{z-4}{4}$$
and parallel to $\overline{\mathbf{i}} - \overline{\mathbf{k}}$.