Emerging Scholars
University of Texas

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WORKSHEET 33

1. Compute the following integrals

a) $\displaystyle\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{2-x^2}}\,dx$ b) $\displaystyle\int \frac{dx}{7+3x^2}$ c) $\displaystyle\int \frac{x\,dx}{\sqrt{1-4x^4}}$

2. a) Sketch a graph of the function $f(x) = \dfrac{1}{1+x^2}$.

   b) Find the area between the graph and the x-axis from $x = -1$ to $x = 1$.

   c) Find the area between the graph and the x-axis from $x = -100$ to $x = 100$.

   d) Let M be a very large number. Find the area between the graph and the x-axis from $x = -M$ to $x = M$.

   e) What happens to the area as $M \to \infty$? What is the meaning of this in terms of your graph?

3. A billboard k feet wide is perpendicular to a straight road and is s feet from the road. At what point on the road would a motorist have the best view of the billboard; that is, at what point on the road is the angle subtended by the billboard a maximum?

4. Find the fallacy in the following argument that $0 = 1$.

$$u = \frac{1}{x} \qquad dv = dx$$
$$du = -\frac{1}{x^2}dx \qquad v = x$$

So integration by parts applied to $\int 1/x\,dx$ yields:

$$0 + \int \frac{dx}{x} = \left(\frac{1}{x}\right)x - \int \left(\frac{-1}{x^2}\right)x\,dx$$
$$= 1 + \int \frac{dx}{x}.$$

Thus $0 = 1$.

5. One of the most important functions in analysis is the *gamma function*,

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt, \qquad x > 0.$$

a) Use integration by parts to prove that $\Gamma(x+1) = x\Gamma(x)$.

b) Show that $\Gamma(1) = 1$. Conclude that $\Gamma(n) = (n-1)!$ for all natural numbers n.

The gamma function provides a simple example of a continuous function which **interpolates** the values of $n!$ for natural numbers n.