Emerging Scholars
University of Texas

Fall 1997

WORKSHEET 31

1. Determine the domain and range of each of the following functions:

   a) $f(x) = \sin^{-1} x$ b) $g(x) = \sin^{-1} x^2$ c) $h(x) = \sin^{-1}(x^2 - 1)$

2. Determine whether each of the following equations is true or false. If true, give the reason it is true. If false, give a counterexample to the statement.

   a) $\sin^{-1}(\sin x) = x$ for all x.

   b) $\sin(\sin^{-1} x) = x$ for all x.

   c) $\sin^{-2} x = \dfrac{1}{(\sin x)^2}$ for $x \neq n\pi$ where n is an integer.

   d) $\sin^{-1} x = \dfrac{1}{\sin x}$ for $x \neq n\pi$ where n is an integer.

   e) $\tan(\sin^{-1} x) = \dfrac{1}{\sqrt{x^2 - 1}}$ when $-1 < x < 1$.

3. Either use the equation $\sin^2\theta + \cos^2\theta = 1$ or draw a right triangle to simplify the following:

   a) $\cos(\sin^{-1} x)$ b) $\sec^2(\tan^{-1} x)$ c) $\tan(\sec^{-1} x)\sin(\sec^{-1} x)$ d) $\cot(\tan^{-1} x)$

   WARNING: Be careful about choosing $+$ or $-$ roots! You will need to use the fact that the range of $\sin^{-1}$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the range of $\tan^{-1}$ is $(-\frac{\pi}{2}, \frac{\pi}{2})$, and the range of $\cos^{-1}$ is $[0, \pi]$.

4. Differentiate the equation $f(f^{-1}(x)) = x$ for each of the six trig functions to find the derivatives of the six inverse trig functions

   $\sin^{-1} x$ $\cos^{-1} x$ $\tan^{-1} x$

   $\csc^{-1} x$ $\sec^{-1} x$ $\cot^{-1} x$

5. Use the given trig substitution to find the given antiderivative:

   a) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx; \quad x = \sin\theta$ b) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx; \quad x = \cos\theta$

   c) $\displaystyle\int \frac{1}{1+x^2}\,dx; \quad x = \tan\theta$ d) $\displaystyle\int \frac{1}{1+x^2}\,dx; \quad x = \cot\theta$

   e) $\displaystyle\int \frac{1}{|x|\sqrt{x^2-1}}\,dx; \quad x = \sec\theta$ f) $\displaystyle\int \frac{1}{|x|\sqrt{x^2-1}}\,dx; \quad x = \csc\theta$

6. a) Let $f(x) = \arcsin x + \arccos x$. Give two separate arguments to show that f is a constant. What is that constant?

   b) Lef $f(x) = \arcsin(\cos x)$ for $0 \le x \le \pi$. Show that $f(x) = ax + b$ for constants a and b. Find them.

7. a) Using the identity $\sin(2y) = 2\sin y\cos y$, show that

   $$2\arcsin x = \arcsin(2x\sqrt{1-x^2}). \qquad (*)$$

   Hint: Let $y = \arcsin x$.

   b) Plug in $x = 1$ into $(*)$.

   c) For what values of x is $(*)$ valid? Why?