Emerging Scholars
University of Texas

Fall 1997

WORKSHEET 30

1. Let $f(x) = x(1-x)$. Break $[0,1]$ into four equal intervals.

   a) Use these to find an upper sum for $\int_0^1 x(1-x)\,dx$.

   b) Use these to find a lower sum for $\int_0^1 x(1-x)\,dx$.

   c) Find the average of the previous two answers.

   d) WITHOUT computing $\int_0^1 x(1-x)\,dx$ show that it is within 1/16 of the average you computed in part c).

   Hint: plot the upper sum, lower sum, and average on a number line. What do you know about where the actual integral lies? Who won the Nobel Peace Prize this year?

   e) Compute $\int_0^1 x(1-x)\,dx$.

2. Let $f(x) = x^2$.

   a) Into how many equal intervals must you subdivide $[0,1]$ for the average of the upper and lower sums to accurately estimate $\int_0^1 x^2\,dx$ to two decimal places?

   b) Compute the upper and lower sums for this partition and find the average.

   c) Compute $\int_0^1 x^2\,dx$. To how many decimal places was your estimate in part b) correct?

3. a) Suppose that $f'(x) = f(x)g'(x)$ for some g. Show that $f(x) = Ke^{g(x)}$ for some K.

   b) Newton's Law of Cooling states that the rate of change of the temperature T of an object is proportional to the difference between T and the temperature τ of the surrounding medium. A cup of coffee at 200° in a room of temperature 70° is stirred continually and reaches 100° after ten minutes. At what time was it at 120°?

4. Your well-intentioned roommates wake you up by bringing you a fresh cup of coffee. What they didn't know is that you ALWAYS take a shower before drinking your coffee. Also, you drink your coffee with milk in it. If you want your coffee to be as hot as possible when you get out of the shower, should you (a) go to the refrigerator and put milk in it now or (b) wait until you get out of the shower to do this?

5. Two identical ice trays are filled, one with two cups of water at room temperature and the other with two cups of boiling water. Both are placed in the freezer. Which tray has solid ice first?

6. The half-life of a radioactive material is the number of years required for 1/2 of the atoms in a sample to decay.

   a) Suppose that after one year, only 36.79% of an initial amount of radioactive material remains. Find the half-life.

   b) From the information given, can you compute the initial amount present?

7. A year ago, there were 4 grams of a radioactive substance. Now there are 3 grams. How much was there 10 years ago?

8. **Carbon-14 Dating.** The half-lives of radioactive elements can sometimes be used to date events from the Earth's past. The ages of rocks more than 2 billion years old have been measured by the extent of the radioactive decay of uranium (half-life 4.5 billion years!). In a living organism, the ratio of radioactive carbon stays fairly constant during the lifetime of the organism, being approximately equal to the ratio in the organism's surroundings at the time. After the organism's death, however, no new carbon is ingested, and the proportion of carbon-14 in the organism's remains decreases as the carbon-14 decays. Since the half life of carbon 14 is known to be about 5700 years, it is possible to estimate the age of organic remains by comparing the proportion of carbon-14 they contain with the proportion assumed to have been in the organism's environment at the time it lived. Archeologists have dated shells (which contain $CaCO_3$), seeds, and wooden artifacts this way.

   a) Find k in the equation $\frac{dy}{dt} = ky$ for carbon-14.

   b) What is the age of a sample of charcoal in which 90% of the carbon-14 has decayed?

   c) The charcoal from a tree killed in the volcanic eruption that formed Crater Lake in Oregon contained 39.2% of the carbon-14 found in living matter. About how old is Crater Lake?