

**WORKSHEET 29**

1. Let  $f(x) = 1/x$  be defined on  $[1, 2]$ . Break the interval into five equal intervals. Use these to find an upper sum for

$$\int_1^2 \frac{1}{x} dx.$$

2. a) Suppose that on some interval, the function  $f$  satisfies  $f'(x) = f(x)$ . Show that  $f(x) = Ke^x$  for some  $K$  by considering the function  $g(x) = \frac{f(x)}{e^x}$ .  
 b) Find the derivative of  $f(x) = e^{x-5}$ . Does this contradict your work from part a)?  
 c) Suppose that on some interval, the function  $f$  satisfies  $f'(x) = cf(x)$  for some number  $c$ . Show that  $f(x) = Ke^{cx}$  for some  $K$  by considering the function  $g(x) = \frac{f(x)}{e^{cx}}$ .  
 d) Suppose that  $f'(x) = f(x)g'(x)$  for some  $g$ . Show that  $f(x) = Ke^{g(x)}$  for some  $K$ .

3. Compute the following integrals

$$\text{a) } \int \frac{e^{2x} + e^x}{e^x} dx \quad \text{b) } \int x(x+1)^2 dx \quad \text{c) } \int \cos x \sqrt{\sin x} dx \quad \text{d) } \int \frac{x^2 - 1}{x+1} dx$$

4. **Newton's Law of Cooling** states that the rate of change of the temperature  $T$  of an object is proportional to the difference between  $T$  and the temperature  $\tau$  of the surrounding medium. A cup of coffee at  $200^\circ$  in a room of temperature  $70^\circ$  is stirred continually and reaches  $100^\circ$  after ten minutes. At what time was it at  $120^\circ$ ?

Hint: Consider Problem 2a).

5. Show that

$$\int_{-\pi}^{\pi} \frac{7x^3 - 22x}{x^{50} + \cos x} dx = 0.$$

6. Find the following integrals:

$$\text{a) } \int_0^{\pi/4} \sec x \tan^2 x dx \quad \text{b) } \int_1^e \frac{1}{x\sqrt{\ln x}} dx \quad \text{c) } \int_0^{\pi} \frac{\sin x}{\cos^2 x} dx$$

7. Let  $f(x) = x(\ln x)^2$  on the interval  $[e^{-4}, e]$ .

- a) Differentiate and simplify.  
 b) Find all points  $c$  where  $f'(c) = 0$  or  $f'(c)$  does not exist.  
 c) Use the first derivative test to classify the above points.  
 d) Find all local and global extrema.

8. Solve for  $x$  if

$$\frac{e^x}{\sqrt{e^x}} = 2.$$