

WORKSHEET 28

1. Find all functions which are their own derivative. Graph 4 different ones on the same axes.
2. In lecture, Dr. Davis noted that the expression $\frac{\ln(x+1)}{\ln x}$ cannot be simplified. Nevertheless, solve

$$\frac{\ln(x+1)}{\ln x} = 2.$$

3. Let $f(x) = \ln x$ and $g(x) = \ln|x|$.

a) Graph f and g on separate axes.

b) Given that $f'(x) = \frac{1}{x}$, prove that $g'(x) = \frac{1}{x}$.

4. Find the derivatives of each of the following. Simplify your answer as much as possible.

a) $y = \sin(e^{x^2+x-1})$

b) $y = \sqrt{1 + e^x}$

c) $y = a^x x^a, \quad a > 0 \quad a, \text{ constant}$

d) $y = 2^{(3^x)}$.

5. Find all vertical and horizontal asymptotes of the following functions:

a) $f(x) = \ln(x+2) - \ln(x+1)$ b) $g(x) = \frac{\ln x + 5}{\ln x^2 - 5}$ c) $h(x) = e^{-1/x^2}$

6. Given that you know $\frac{d}{dx}[e^x] = e^x$, prove the following:

a) $\frac{d}{dx}[a^x] = a^x \ln a$ b) $\frac{d}{dx}[a^u] = a^u \ln a \frac{du}{dx}$ c) $\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$ d) $\frac{d}{dx}[\log_a u] = \frac{1}{u \ln a} \frac{du}{dx}$

Hint for part c - Set $u = \log_a x$ and use part b.

7. In a certain village, there are 50 couples. As it turns out, everyone in the village is having an affair. This despite a particularly gruesome custom which requires a wife, upon discovering that her husband is having an affair, to kill him the following morning. Even more oddly, the women in the town talk quite freely about their activities! In fact they are all perfectly aware that any woman who is having an affair will tell EVERY other woman in the village except, of course, for the wife of the man with whom they are having the affair. Yet life there goes on quite peacefully since no woman can know for sure that her husband is actually having an affair. One day, a well revered wise man visits the village and announces that someone is having an affair. What happens after this?