

WORKSHEET 26

1. Find the domain and range of the following functions.

a) $f(x) = \ln x^2$ b) $g(x) = 2 \ln x$ c) $r(x) = \ln |\sin x|$ d) $s(x) = \ln(\sin x)$.

2. When, if ever, are functions f and g from Problem 1 equal?

3. Review the properties of logs with your group* and then solve these equations for x :

a) $x = \log_4 2$

f) $3 \log_x 4 = 2$

b) $\log_4 x = 5/2$

g) $\log_2 x = \log_4 5 + 3 \log_2 3$

c) $\log_x \frac{1}{8} = -\frac{3}{2}$

h) $\log_x(1 - x) = 2$

d) $\log_4(x - 2) - \log_4(2x + 3) = 0$

i) $\log_8(\log_4(\log_2 x)) = 0$

e) $\log_{10} x + \log_{10}(x - 15) = 2$

*If you are having trouble remembering the properties of logs, see Problem 9.

4. Show that $\log_a b = \frac{1}{\log_b a}$.

5. Suppose you place a penny on one corner square of a chess board, two on the next square, four on the next, eight on the next, and so on. How tall will the stack of pennies on the last square be? (There are eight squares per side on a chess board.)

6. Use a calculator to estimate

$$\lim_{s \rightarrow 0} (1 + s)^{1/s}.$$

This limit exists and its value is so important in mathematics (and in any field that uses mathematics) that we have given it a name, e . (What other constants in mathematics have their own letter?)

7. Let $f(x) = \log_{10} x$. Use the definition of the derivative to compute $f'(x)$.

Hint: Use the properties of logarithms to manipulate the difference quotient until you can evaluate the limit. Take special note of the definition of e given in Problem 6.

8. a) For what values of b is $f(x) = \log_b x$ an increasing function?

b) Sketch the functions $\log_e x$ and $\log_{1/e} x$.

c) For what values of b is $f(x) = b^x$ an increasing function?

d) Sketch the functions e^x and $(1/e)^x$.

9. Three useful properties of exponentials are given below

(note: they are true in any positive base as well as in base 10).

$$10^x 10^y = 10^{x+y}$$

$$\frac{10^x}{10^y} = 10^{x-y}$$

$$(10^x)^r = 10^{rx}$$

a) Letting $a = 10^x$ and $b = 10^y$, rewrite these equations without explicit reference to x or y and without exponentials. What you obtain will be three corresponding properties of logarithms.

b) For what values of a and b are your equations valid?