Emerging Scholars
University of Texas

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WORKSHEET 25

1. a) State the Fundamental Theorem of Calculus.

   b) What is wrong with the following argument?

$$\int_{-1}^{2} \frac{dt}{t^2} = \left.\frac{-1}{t}\right|_{-1}^{2} = -\frac{1}{2} - \left(-\frac{1}{-1}\right) = -\frac{1}{2} - 1 = -\frac{3}{2}$$

2. Find all continuous functions $f(x)$ satisfying

$$\int_0^x f(t)\,dt = [f(x)]^2 + C$$

(Hint: Differentiate both sides with respect to x.)

3. Suppose there is a differentiable function $y = f(x)$ on the interval $[a, b]$. Let $P_1 = (x_k, f(x_k))$ and $P_2 = (x_{k+1}, f(x_{k+1}))$ be points on the graph of this function. Suppose also that $x_{k+1} - x_k = \Delta x$ is a very small quantity.

   a) Show that the length of the line segment P_1P_2 is approximately equal to $\Delta x\sqrt{1 + (f'(x))^2}$.

   b) Write a definite integral which expresses the length of the graph of f on the interval $[a, b]$.

4. Find the length of the curve $y = (1 - x^{2/3})^{3/2}$ from $x = 1/8$ to $x = 1$.

5. Graph $f(x) = 1/x^2$ on your calculator for $-5 \le x < 0$. Notice that as the graph is drawn, it appears to speed up as it approaches $x = 0$. This is because (roughly) your calculator plots the graph by evaluating the function at various values of x evenly spaced throughout the domain and "connecting the dots" as it goes. For x values close to 0, the values of the function are farther apart than they are for values of x near -5. Let's figure out how fast things appear to be moving on your calculator.

   a) Express as an integral the length of the curve from $(-5, f(-5))$ to $(b, f(b))$.

   b) Suppose that your calculator can plot this graph in 1.5 seconds (You may time your own calculator and change this value if it does not seem accurate for your calculator.) Let's assume that the plotting moves left to right (along the x-axis) at a constant rate. How fast is the length of the curve you see on your calculator increasing when $(-3, f(-3))$ is being plotted? when $(-2, f(-2))$ is being plotted? when $(-.5, f(-.5))$ is being plotted?

6. a) Sketch the graph of $f(x) = 4x - x^2$ on the interval $[0, 4]$. Estimate a value which you think is roughly the average height of f over this interval.

   b) Partition the interval $[0, 4]$ into five subintervals (not all the same size). Use an actual value of f on each subinterval to compute an estimate for the average value of f on $[0, 4]$.

   (Note: You must take into account that not all the intervals are the same size, so you must weight each value appropriately.)

   c) Write down the actual average A as a limit of sums. Use this to show that

$$A = \frac{1}{4-0}\int_0^4 4x - x^2\,dx.$$

7. Average Velocity: Two methods.
Suppose an object travels along a straight wire such that (i) it starts at one end of the wire, and (ii) the velocity of the object t seconds later is

$$v(t) = \frac{24}{(t+1)^2} - 4$$

a) Where is the object at time $t = 1$? at $t = 4$? (How did you get your answers? Could you have done it another way?)
b) Use your answer above to determine the average velocity of the object over that time.
c) Now calculate the average velocity with a definite integral.
d) How do your methods compare? Why do they give you the same answer?
e) Does the object ever return to the end where it started? If so, when?