Emerging Scholars
University of Texas

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WORKSHEET 22

1. In physics, the equation $W = Fd$ gives the amount W of energy (or work) expended in moving a distance d by applying a constant force F.
   a) A force of 10 N (Newtons) is applied to move an object 4 cm, then a force of 7 N is applied in moving it 6 more cm. How much energy W was expended?
   b) Now suppose the force over the first cm is 2 N, the force over the second cm is 4 N, and so on. How much energy W is expended over the entire 10 cm?

2. Hooke's law says that a spring requires a force $F = kx$ to hold the end displaced by a distance x from its equilibrium point. Here k is a constant, which depends on how "stiff" the spring is.
   a) Explain why we cannot use the formula $W = fd$ to find the amount of energy required to displace a spring 10 cm.
   b) Describe a way to use the formula $W = Fd$ to find an approximation to the amount of energy required to displace a spring 10 cm. Find an approximation.

3. From physics recall that the force acting on an object is equal to its mass times its acceleration, and that work is equal to the force acting on an object multiplied by the distance the object has traveled ($F = ma$ and $W = Fd$.) In lifting a mass, the force to be overcome is that of gravity which (on the surface of the Earth) corresponds to an acceleration of 9.8 m/s^2.
   a) Suppose a 1 kg bucket filled with 10 kg of water (equals approximately 10 liters - isn't the metric system great?) is lifted 10 m into the air. How much work is done?
   b) Now suppose a bucket of water is lifted 10 meters into the air in such a way that (i) there are 10 kg of water in the bucket at the start, (ii) the bucket is lifted at a constant rate, (iii) the water leaks out of the bucket at a constant rate, (iv) the bucket contains only 5 kg (how many liters?) of water when its is 10 m high.

      Break the 10 m into eight segments of your choosing. For each segment find one value which is a good estimate for the force required over that distance. Use those values for F to estimate the work done over each interval. Add up your eight estimates to obtain an estimate for the total work required to lift the leaky bucket 10 m.

4. Aliens have kidnapped Marisa! She will be set free only if she can beat their "hero" at a game of Gorgo. The game is simple. A large metal bar 120 meters long (metric aliens!) is brought before the contestants. Whoever can guess the weight of the bar most precisely, wins.

   When the day of the contest arrives, Marisa is shoved into a huge arena before millions of screaming aliens. While staring at the long metallic Gorgo bar, she hears a low rumble begin, and a fearful hush falls over the crowd. As a door at the opposite end of the arena lifts, Marisa gasps in horror: it's Elvis.

   Elvis looks at the bar for only a second, mutters something unintelligible, and the crowd breaks out in delirium throwing Twinkies and little blue pills into the arena. Now it is Marisa's turn. She goes to the bar, takes small samplings at 20 meter intervals, and finds the density (in kilograms per centimeter). Here is her data:

Meter mark	Density	
0	30.4	
20	$\cdots$	Data lost. (Stolen by aliens?)
40	46.5	
60	65.8	
80	29.2	
100	52.1	
120	55.0	

   Estimate the total weight of the Gorgo bar.

5. Suppose that a wind-up car is designed so that the velocity t seconds after it is started is given by $v(t) = \sin(\sqrt{\pi t/10})$ (measured in m/sec.) Your task is to estimate the position of the car after 15 seconds. Do this by estimating the velocity over each one-second time interval by an appropriate constant velocity.