

WORKSHEET 20

- Suppose for some function $h(x)$, we know that $h(0) = 2$ and that $h'(x) = (1 + x^3)^{1/3}$. Draw the graph of $h(x)$.
- The mathematicians at Los Alamos Laboratory developed the following equation to describe the change over time in the number of people infected with HIV.

$$\frac{dI}{dt} = \alpha I(t) \left[1 - \frac{I(t)}{N} \right]$$

where

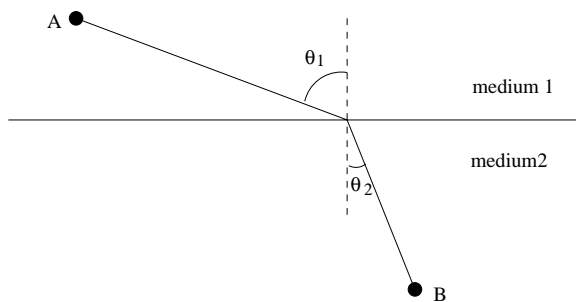
$I(t)$ = number of people infected at time t

N = size of the population

α = rate at which an infected person passes on the virus per unit time.

Assume t is measured in days, $N = 100,000$ and $\alpha = 0.01$.

- Is the number of infected people increasing or decreasing?
 - How many people have to be infected for the number of people infected to stop increasing?
 - How many people are being infected per day when there are 100 people infected?
 - Find $\frac{d^2 I}{dt^2}$ using implicit differentiation then write it as a function of $I(t)$.
 - Is the number of infected people accelerating, or increasing at a slower rate and slower rate?
 - How many people have to be infected for the number of people infected to stop accelerating?
- Snell's Law** Fermat's Principle in optics says that light travels from point A to point B along the path that requires the least amount of time. Suppose that light travels in one medium at velocity c_1 and in a second medium at velocity c_2 . If A is in medium 1 and B is in medium 2, and the x -axis separates the two media, as in the figure below, show that $\frac{\sin \theta_1}{c_1} = \frac{\sin \theta_2}{c_2}$.



- Suppose you know that $f(x) = \frac{2}{1+x^2} - \frac{3}{4+x^2}$. Is $f(0) < f(1)$?
- A certain function f defined for all x has

$$\begin{aligned} f''(0) &= 0, \\ f''(x) &> 0 \quad \text{for } x > 0, \\ \text{and } f''(x) &< 0 \quad \text{for } x < 0. \end{aligned}$$

The number of critical points, number of local maxima, number of local minima, and number of roots of f are all tabulated. Give all possible such tabulations.