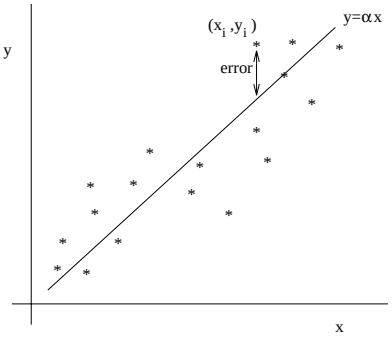
Emerging Scholars
University of Texas

Fall 1997

WORKSHEET 19

1. Suppose we want to fit a line to a set of data as shown in the figure to the right. The *least squares method* is based on minimizing the sum of the squares of the errors. If we have a set of data points (x_i, y_i) for $i = 1, 2, \ldots, n$ this will be a sum of n positive values. The error for a data point (x_i, y_i) is the difference between y_i and the height at x_i of the line we select.



   a) Suppose we choose a line of the form $y = \alpha x$. Find an expression for $S(\alpha)$, the sum of the squares of the errors.

   b) Find a formula for α which minimizes $S(\alpha)$.

2. A certain function f defined for all x has

$$\begin{aligned} f''(0) &= 0, \\ f''(x) &> 0 \quad \text{for } x > 0, \\ \text{and } f''(x) &< 0 \quad \text{for } x < 0. \end{aligned}$$

   The number of critical points, number of local maxima, number of local minima, and number of roots of f are all tabulated. Give all possible such tabulations.

3. In this problem, you are going to trace the shape of your hand and approximate the area of the picture that you create. Your main tasks are to devise a method for approximating the area and to show that your approximation is very close to the actual area.

   a) Start with a sheet of blank paper and make an accurate trace of the outline of your hand.

   b) Devise a method to approximate the area of the region inside the curve you have traced. You must explain your method in detail and why it works.

   c) Find a way for estimating the error for the method you devised.
   Note: Error is something you would like to make *small*! Thus an estimate for the error means being able to say the error is **less than** some value.

   d) Find an approximation off the area of your tracing that differs from the actual area by less than 1% of the actual area.

4. Let $f(x) = (x^2 - 4)/(x - 1)$

   a) Find the intercepts.

   b) Find all vertical asymptotes.

   c) Differentiate and simplify.

   d) Find all points c where $f'(c) = 0$ or $f'(c)$ does not exist.

   e) Use the first derivative to find where f is increasing and decreasing.

   f) Find the second derivative of f and simplify.

   g) Find all points c where $f''(c) = 0$ or $f''(c)$ does not exist.

   h) Use the second derivative to find where f is concave up and concave down.

   i) Plot all intercepts, places where $f'(c) = 0$, where $f''(c) = 0$ and all asymptotes.

   j) Graph f.