

### WORKSHEET 18

#### Chain Rule

1. Suppose that  $f$  and  $g$  are two functions such that  $f(g(x)) = x$ . Show that

$$g'(x) = \frac{1}{f'(g(x))}.$$

2. Let  $y = \tan^2(\arccos(x))$ .
- Find  $dy/dx$  using the chain rule.
  - Find another expression for  $y$  which does not mention trig functions. (Hint: Draw a triangle.)
  - Find  $dy/dx$  from your expression in part b). How does it compare with your answer in part a)?

#### Implicit Differentiation

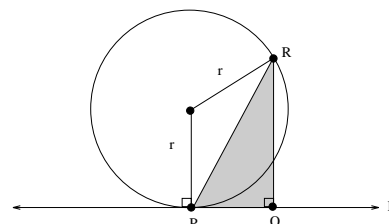
3. A differentiable function  $y(x)$  satisfies  $x^2 \cos y + \sin y = x$  and  $y(1) = 0$ . What is  $y'(1)$ ?
4. Prove that the tangent line to a circle at any point is perpendicular to the radius to that point.

#### Related Rates

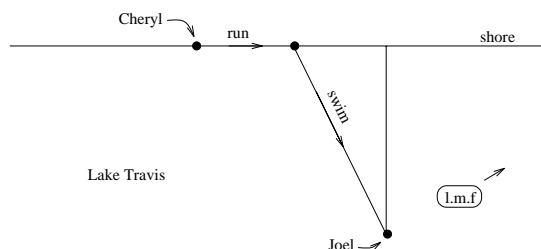
5. Given that a spherical raindrop evaporates at a rate proportional to its surface area, how fast does the radius shrink?
6. A boat is pulled in to a dock by means of a rope with one end attached to the bow of the boat, the other end passing through a ring attached to the dock at a point 4 feet higher than the bow of the boat. If the rope is pulled in at a rate of 2 feet per second, how fast is the boat approaching the dock when
- 10 feet of the rope are out?
  - 5 feet of rope are out?

#### Max/Min Problems

7. Suppose you are given a circle of radius  $r$  and a tangent line  $L$  to the circle through a point  $P$  on the circle. From a variable point  $R$  on the circle, a perpendicular  $PQ$  is drawn to  $L$  with  $Q$  on  $L$ . Determine the maximum of the area of triangle  $PQR$ .



8. Joel has fallen off of his "Little Mermaid" floaty 200 feet from shore in Lake Travis. Cheryl is at a point 200 feet down the shore from the point closest to Joel. She can run 18 ft/s and can swim at a rate of 5 ft/s.
- To what point on the shore should she run before diving into the lake **if** she wants to reach Joel as quick as possible?



- Once Joel falls into the water, he can manage to thrash about for one minute. It takes Cheryl 10 seconds to notice Joel is in trouble. Can she reach him in time?