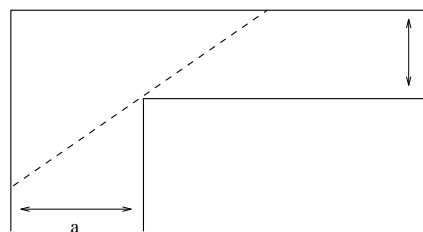


### WORKSHEET 17

1.
  - a) Find the dimensions of a can made of aluminum, holding 12 ounces of soft drink, yet using the least amount of aluminum.
  - b) Suppose the 12-ounce can is constructed so that the top and bottom of the can has a manufacturing cost which is twice that of the side. What dimensions should the can have to minimize the cost of the can?
  - c) Measure the diameter and height of a standard 12-ounce can. Based on these measurements, what is the ratio of the cost of the ends (top and bottom) of the can to the side which makes its dimensions a solution to the problem of minimizing the cost of the can?
  - d) A 12-ounce can is to be manufactured. There is no waste involved in cutting the metal that makes the vertical sides of the can, but each end piece is to be cut from a square of metal and the corners of the square are wasted. Find the dimensions of the most economical can.
2. Twenty feet of wire are to be used to form two figures. In each of the following cases, how much should be used for each figure so that the total enclosed area is a maximum?
  - a) equilateral triangle and square
  - b) square and regular pentagon
  - c) regular pentagon and hexagon
  - d) regular hexagon and circle

(Hint: The area of a regular polygon with  $n$  sides of length  $x$  is  $A = \frac{n}{4} \cot(\frac{\pi}{n})x^2$ .)  
What can you conclude from this pattern?

3. Two hallways meet at right angles. Their widths are  $a$  and  $b$  as indicated in the picture. What is the greatest length of a ladder which can be carried horizontally around the corner?



4. Here is a scenario which occurred many millenia ago: The patriarch of a wealthy family was on his deathbed and wanted to divide his gold among his eight sons. They were all very very greedy and completely mistrusted each other. Wishing to favor the oldest son (as tradition would have it) but also to reward the more cunning of his progeny, he made the following decree:

The oldest son is to propose a plan for dividing up the gold. The sons are all to vote on this plan, and if it receives at least half of the votes (four or more) then that will be the way the gold is divided. If this plan does not receive half of the votes, the oldest son gets nothing, the next oldest proposes a plan, and there is another vote, now among the remaining seven. Again at least half of the vote (still four or more) is required, and failure removes this son from the process. This is to continue until some son's plan receives at least half of the votes of the remaining heirs.

Keep in mind that there is absolutely no trust among the eight sons and each one's main priority is obtaining the most gold possible for himself.

How much gold (if any) will the oldest son be able to inherit?