Emerging Scholars
University of Texas

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WORKSHEET 15

1. In each of the following equations, suppose that each variable is actually a function of time t and differentiate each expression with respect to t.

   a) $x^2 + y^2 = 100$ b) $\dfrac{x+s}{5} = \dfrac{s}{1.5}$ c) $40y - xy = 80$

   d) $(x+7)(7-gt^2) = 9x$, where g is a constant e) $V = kh^3$, where k is a constant

2. One sweltering 108° day this past August, Josh and Joel were cleaning the gutters of their elderly neighbor, Mrs. Macinac, in repentence for having earlier hit a baseball through her dining room window. While Joel was perched atop a 10 foot ladder, he made the mistake of angering Josh by accusing him of sleeping on the job. In retaliation, Josh began to pull the base of the ladder away from the wall at a rate of $\frac{1}{2}$ ft/sec.

   For the following questions, assume that Joel's balance is very good, and that the ladder was originally flat against the wall.

   a) How far does Joel fall during his first four seconds of motion? The next four? The next four? The next four? The last four? (Use a calculator.)

   b) From what you found in part a), what can you say about the rate at which Joel is falling?

   c) How fast is Joel approaching the ground when Josh has pulled the bottom of the ladder 6 feet from the wall.

   d) How fast is Joel moving when he hits the ground? What is the physical significance of your answer?

   e) The ladder, the wall, and the ground form a triangle. How fast is the area of the triangle changing when Joel is 8 feet from the ground? Is the triangle getting larger at this time, or smaller?

3. a) A streetlight hangs 5 meters above the ground. Regina, who is 1.5 meters tall, walks away from the point under the light at a rate of 2 meters per second. How fast is her shadow lengthening when she is 7 meters away from the point under the light?
   (Hint: Use similar triangles.)

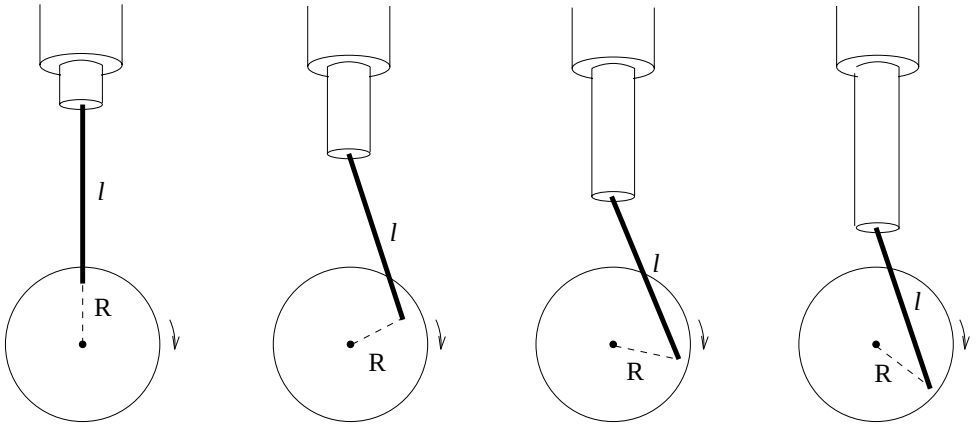
   b) Suppose Regina has the ability to magically shrink herself. At what rate must she do this to keep her shadow a constant length of 3 meters? Write this as a function of only her distance from the point under the light.

   c) A light is on the ground 40 meters away from a building. Chris, who is 2 meters tall, walks from the light toward the building at 2 meters/second. How rapidly is his shadow on the building growing when he is 20 meters from the building?

   d) A light is at the top of a pole which is 9 meters high. A ball is dropped from height 7 at a point which is at a horizontal distance 7 meters from the pole. Assume that the ball falls according to the law $s = gt^2$, where t is the time in seconds, s is the distance in meters, and $g = 4.9$ is a constant. Find how fast the shadow of the ball is moving along the ground at the time 4 seconds after it is dropped.

4. Sand is flowing from a pipe at the constant rate of s cubic meters per second, and is falling in a conical pile. The diameter of the base of this pile is always three times the altitude. At what rate is the altitude of the pile increasing when the altitude is h meters?

5. Many devices turn circular motion into linear motion or vice versa by employing a version of the following device. One end of a rod is connected to a piston which is in a compartment which only allows linear motion. The other end of the rod is connected near the outside of a wheel which is free to rotate around its center. A diagram of such a system is pictured below at four different times. (This might be pistons and driveshaft in an automobile or the foot pedal and wheel of a spinning wheel or some similar manually powered device, for examples.)



In the following suppose the rod in the diagram above has lenght l and is attached to the wheel at a point R units from its center.

a) If the wheel spins at a constant rate and goes around twice every second, how fast is the piston moving when it is half way between its highest and lowest points?

b) Suppose $l = 8$ and $R = 2$, and that at time 0 the piston is at its highest point. Come up with a function which gives the velocity of the piston t seconds later. (What does the sign of you velocity indicate? How?)

c) Now come up with a function which gives the position of the piston t seconds later. (What did you pick for the height 0?)

d) When is the piston moving fastest?