Emerging Scholars
University of Texas

Fall 1997

WORKSHEET 14

INSTRUCTIONS

Do not answer the questions asked in the related rates problems given below.
Instead, follow these instructions:

a) Draw **and label** a picture of the situation.

b) The rate(s) you know and the rate you are seeking should be the time derivatives of quantities you have labeled. State what those quantities are.

c) Determine, an algebraic relationship involving the quantities you have identified.

d) Finally, venture a guess as to what type of answer you would get. Will it be positive or negative? How would the rate depend on the variables in the problem?

PROBLEMS

1. Imagine the following magic triangle. Its base is on a horizontal surface and no matter what you do to its height, the triangle always has area 10. If you push down on the top of the triangle so that it becomes shorter at a rate of 3 cm/sec, how fast will the length of the base be changing when the triangle is 5cm tall?

2. Sophie and Elisa have made themselves two dimensional! Sophie moves along the positive horizontal axis, and Elisa along the graph of $f(x) = -\sqrt{3}x$, $x \leq 0$. At a certain time, Sophie is at the point $(5, 0)$ and moving with speed 3 units/sec; and Elisa is at a distance of 3 units from the origin moving with speed 4 units/sec. At what rate is the distance between Sophie and Elisa changing?

3. For the following problem, assume that Ben is perfectly spherical. Suppose also that Ben melts at a rate proportional to his surface area a (i.e., $\frac{dv}{dt} = ka$ for some negative constant k.) how fast is Ben's radius changing when his radius passes the 3 cm mark? when his radius is 5 cm? when his radius is r cm? (your answers might involve the constant k.)

4. Rain is falling at the rate of q inches per hour into an open conical tank of height h and radius r. show that at each instant the rate at which water is rising in the tank is

$$q \times \frac{\text{(area of tank openening)}}{\text{(area of water surface)}}.$$

   next show that this holds for an open tank of arbitrary shape.

e. Amanda and Amanda are on a ferris wheel when the sun is directly overhead. the diameter of the wheel is 50 feet, and its speed is 0.1 revolution per second. (i) what is the speed of the Amandas' shadow on the ground when they are at a two-o'clock position? (ii) a one-o'clock position? (iii) show that their shadow is moving fastest when they are at the top or bottom, and its slowest when they are at the three-o'clock and nine-o'clock positions.

f. A two-piece extension ladder leaning against a wall is collapsing at the rate of 2 feet per second at the same time as its foot is moving away from the wall at the rate of 3 feet per second. How fast is the top of the ladder moving down the wall when it is 8 feet from the ground and the foot is 6 feet from the wall?

g. The speed limit on a stretch of highway is 55 mph. Highway patrol officer, Sgt. Miguel, stations himself at a point, out of view of the motorists, 50 feet off the highway. Miguel is equipped with a radar gun which measures the speed at which a car approaches **his position**. He takes a reading of suspected speeders by pointing his radar gun at a point on the highway 120 feet from the point on the highway closest to him. The radar gun picks up a reading of 48 feet/sec for a green Chevy driven by Alyssa. How fast is she traveling? Is Alyssa speeding?

WAIT! You don't get to leave yet!

MORE INSTRUCTIONS

e) Differentiate the expression you found in part c) with respect to time t.
Plug in the appropriate values given for any variables or rates then solve for the requested rate.