Emerging Scholars
University of Texas

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WORKSHEET 13

1. When you see the function $\theta \mapsto \sin\theta$ it is assummed that θ is given in radians. Define a **different** function f to by setting $f(x)$ as the value of the sine of an angle which measures x **degrees**. Let g be defined similarly for cosine.

   NOTE: These are indeed different functions! For most values of x, $f(x) \neq \sin x$. Why not?

   a) Express $f(x)$ and $g(x)$ in terms of sin and cos.
   Remember: x has units of degrees and sin and cos want to be given radians!

   b) What is $\frac{df}{dx}$? What is $\frac{dg}{dx}$? (Hint: Use part a) and the chain rule.)

   c) Express $\frac{df}{dx}$ and $\frac{dg}{dx}$ in terms of $f(x)$ and $g(x)$. (No mention of sin or cos allowed.)

   d) Is it still true that $(f(x))^2 + (g(x))^2 = 1$?

   e) Why don't we use the unit of degrees in calculus?

2. a) Solve each of the following equations for y, then find $\frac{dy}{dx}$.

   $x^{1/2} + y^{1/2} = 1$ $\qquad |x| + |y| = 1$ $\qquad x^2 + y^2 = 1$ $\qquad x^3 + y^3 = 1$

   b) Use implicit differentiation to find $\frac{dy}{dx}$.

3. Consider the relation between x and y given by

   $$(\cos x)y^2 + (3\sin x - 1)y + (7x - 2) = 0.$$

   a) Check that the pair of values $x = 0$ and $y = 2$ satisfy this relation.

   b) Find y' at the point $(0, 2)$ using implicit differentiation.

   c) Explicitly find an equation for y in terms of x which will be valid near the point $(0, 2)$.
   (Hint: When solving quadratic equations, it is sometimes the **most** difficult part of the problem to decide which of $\pm$ to use!)

   d) Would you like to find $\frac{dy}{dx}$ at $x = 0$ by differentiating the expression found in c)? Notice that you are **not** actually asked to do the computation following from c).

4. a) For each of the following equations, find $\frac{dy}{dx}$:

   i) $x^2 + y^2 = 1$ $\qquad$ ii) $y = \cot^2 x$

   b) For each equation above, find $\frac{dx}{dy}$. Explain the difference between the **meaning** of this derivative and the one you found in part a).

   c) Now suppose that x and y are both dependent on the variable t (perhaps t denotes time). For each equation above, suppose also that these functions $x(t)$ and $y(t)$ satisfy the equation for all times t. Differentiate each expression with respect to t. Solve for **both** $\frac{dx}{dt}$ and $\frac{dy}{dt}$. Explain the **meanings** of these derivatives.

5. Suppose a particle is moving along a circle described by the equation $x^2 + y^2 = 1$ as in part a) of the previous problem. Here the unit length is one meter, and time t is measured in seconds.

   a) Which derivative expresses the rate (in m/s) at which x is changing?

   b) Which derivative expresses the rate (in m/s) at which y is changing?

   c) Suppose the particle is moving through the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ with horizontal velocity 4 m/s. What is the vertical velocity of the particle?

6. A rectangular room measures 30 feet in length and 12 feet in height, and the ends are 12 feet in width. A fly, with a broken wing, rests at a point one foot down from the ceiling at the middle of one end. A smudge of food is located one foot up from the floor at the middle of the other end. The fly has just enough energy to *walk* 40 feet. Show that there is a path along which the fly can walk that will enable it to get to the food.