

**WORKSHEET 10**

**Mike's Bizarre Function Zoo**

1. a) Graph  $f(x) = \sqrt[3]{x}$  and its tangent line at  $x = 0$ .  
 b) Find the slope of a secant line to  $f$  intersecting the graph when  $x = 0$  and  $x = h$ .  
 c) Find the limit of these slopes as  $h \rightarrow 0$ .  
 d) Find  $\lim_{x \rightarrow 0} f'(x)$ .  
 e) What is the equation of the tangent line to the graph of  $f$  at  $x = 0$ ?
  
2. Let  $s(x) = \begin{cases} e^{-1/x^2} & x \neq 0; \\ c & x = 0, \end{cases}$  where  $c$  is a constant.  
 a) Find the value of  $c$  for which  $s$  is continuous.  
 b) Use your calculator to compute the limit  $s'(0)$ .
  
3. Let  $g(x) = \begin{cases} x \sin(1/x) & x \neq 0; \\ a & x = 0, \end{cases}$  and  $h(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0; \\ b & x = 0. \end{cases}$   
 a) Find the values of  $a$  and  $b$  for which  $g$  and  $h$  are continuous functions.  
 b) Find the limits  $g'(0)$  and  $h'(0)$ .  
 c) Sketch graphs of  $g$  and  $h$ . Explain geometrically your results to part b).
  
4. Let  $r(x) = \begin{cases} 1 & x \text{ is rational;} \\ 0 & x \text{ is irrational.} \end{cases}$   
 a) Sketch a graph of  $r$ . Is it continuous? Why or why not?  
 b) Sketch several secant lines through  $x = 0$  and  $x = h$  where  $h$  is very small. Find the limit  $r'(0)$ .
  
5. Let  $v(x) = \begin{cases} x^2 & x \text{ is rational;} \\ 0 & x \text{ is irrational.} \end{cases}$   
 a) Sketch a graph of  $v$  and show that it is continuous at  $x = 0$ .  
 b) Show that  $v$  is differentiable at  $x = 0$ .
  
6. Let  $w(x) = \begin{cases} 1/q & x \neq 0 \text{ is rational and } x = p/q \text{ is in simplest form;} \\ 0 & x = 0; \\ 0 & x \text{ is irrational.} \end{cases}$   
 a) Fill in the following chart:  

$x$	0	3	3.1	3.14	3.141	3.1415	$\pi$	1	1.4	1.41	1.414	$\sqrt{2}$	2	2.7	2.71	2.718	$e$
$w(x)$																	

  
 b) Sketch a graph of  $w$ . Where is it continuous?  
 c) Where is  $w$  differentiable?