Emerging Scholars
University of Texas

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WORKSHEET 9

1. For each of the following descriptions of a function f, sketch the graph of such a function. If no such function exists, explain why.

   a) $f'(a)$ exists and f is continuous at a.

   b) $f'(a)$ exists but f is not continuous at a.

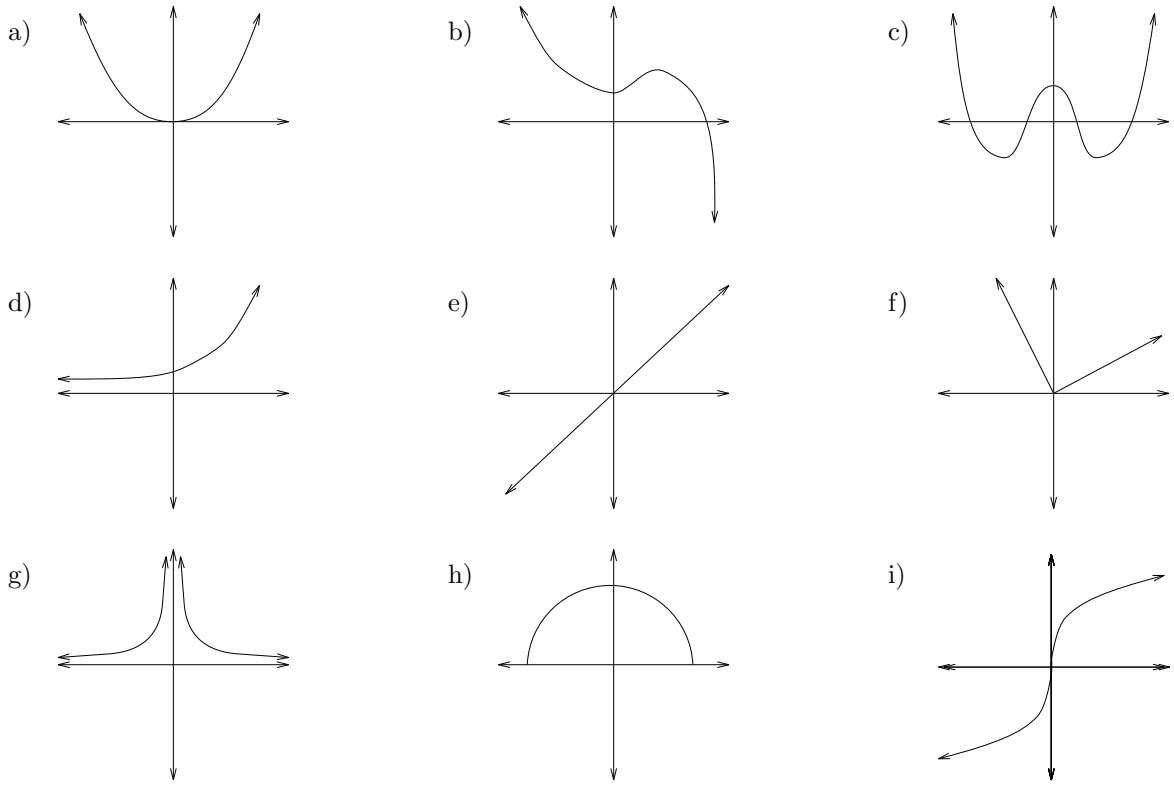
   c) $f'(a)$ does not exist but f is continuous at a.

   d) $f'(a)$ does not exist and f is not continuous at a.

   If any of your examples have a cusp at a, find an example for that description which does not have a cusp.

   If, on the other hand, none of your examples have a cusp at a, indicate which of these four descriptions would be appropriate for such a function.

2. For each graph of a function f below, sketch a graph of its derivative f'.



3. Given below is a *one-parameter family of functions*. That is, it is a collection of many functions f_t - one for each value of the parameter t.

$$f_t(x) = \begin{cases} t\sin x, & x \le \frac{\pi}{2}; \\ (1-t)x + t^2, & x > \frac{\pi}{2}. \end{cases}$$

   a) Determine which values of the parameter t give a continuous function. Graph f_t for each of these values of t.

   b) Determine which values of the parameter t give a differentiable function. Looking at your graphs from part a), give a geometric explanation of your results.

4. The equation of the tangent line to the graph of a function f at $x = 4$ is $y = 3x - 17$.
   a) What is $f(4)$? What is $f'(4)$?
   b) Given that $f(x) = ax^3 + b$, find the constants a and b.

5. Prove starting from the the definition of the derivative and stating any assumptions made about properties of limits (also draw a picture to illustrate):
   a) if $g(x) = f(x) + c$, then $g'(x) = f'(x)$;
   b) if $g(x) = cf(x)$, then $g'(x) = cf'(x)$;
   c) if $g(x) = f(x) + h(x)$, then $g'(x) = f'(x) + h'(x)$.

6. a) Find all points on the graph of $y = x^2$ whose tangent line passes through the point $(5, 0)$.
   b) Show that no line tangent to the graph of $f(x) = x + \dfrac{1}{x}$ passes through the origin.

7. Find the equation to the line tangent to the graph of $y = x^{1/3}$ at the origin.