Emerging Scholars
University of Texas

Fall 1997

WORKSHEET 6

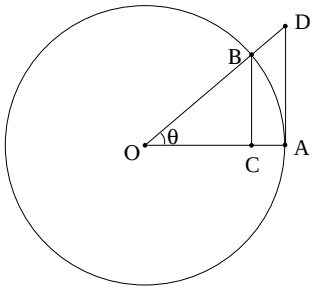
1. Compute the following limits algebraically:

   a) $\lim_{x\to 2} \dfrac{|x-2|}{x^2-2x}$ b) $\lim_{x\to -1^+} \dfrac{|x+1|}{x^3+1}$ c) $\lim_{x\to 2} \dfrac{(x-2)^2}{|x-2|}$

   Note: These are all previous exam problems!

2. a) In the unit circle to the right, find the area of triangle OBC, the area of triangle ODA, and the area of the sector of the circle OBA all as functions of θ.

   b) Use the Squeeze Theorem to find $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta}$.



3. Use your result from Problem 3 to compute $\lim_{\theta\to 0} \dfrac{1-\cos\theta}{\theta}$ algebraically.
   Hint: multiply by a clever form of one.

4. a) Find the slope of the secant line intersecting the graph of $f(x) = \sin x$ at x and $x+h$.

   b) Simplify your expression using the identity

   $$\sin(A+B) = \sin A\cos B + \sin B\cos A.$$

   c) Compute the limit of these slopes as $h \to 0$.

5. Let $f(x) = \begin{cases} 1 & x \geq 2; \\ x & x < 2. \end{cases}$

   a) Find a formula giving the slope of the secant line intersecting $f(x)$ when $x = 2$ and $x = 2+h$.
   Note: You will have different answers for $h < 0$ and $h > 0$. Why?

   b) Find the limit of these slopes as $h \to 0^-$ and $h \to 0^+$.

6. A cake has dimensions $15'' \times 15'' \times 3''$. It is frosted on the sides and top. How can it be divided into 5 pieces so that each piece has the same amount of cake and the same amount of frosting? What if the dimensions are changed? What about 6 pieces? 7 pieces? n pieces?