Emerging Scholars
University of Texas

Fall 1997

WORKSHEET 5

1. Use algebraic techniques to compute the following limits. Show all your steps!

a) $\lim_{x\to 1} \dfrac{x^3 - 5x^2 + 8x - 4}{x^3 - x^2 - 4x + 4}$

b) $\lim_{x\to 2} \left(\dfrac{1}{x-2} - \dfrac{4}{x^2-4} \right)$

c) $\lim_{x\to 3^-} \dfrac{|x-3|}{9-x^2}$

d) $\lim_{x\to -1} \dfrac{x^4 + x}{x^2 + x}$

Note: These have all appeared on Dr. Davis's previous exams.

2. **The Squeeze Theorem:** Suppose $u(x)$ and $v(x)$ are functions which are defined for $c < x < d$ and which have the properties that

i. $u(x) \leq f(x) \leq v(x)$ for all x near b where $c < b < d$, and
ii. $\lim_{x\to b} u(x) = \lim_{x\to b} v(x) = L$.

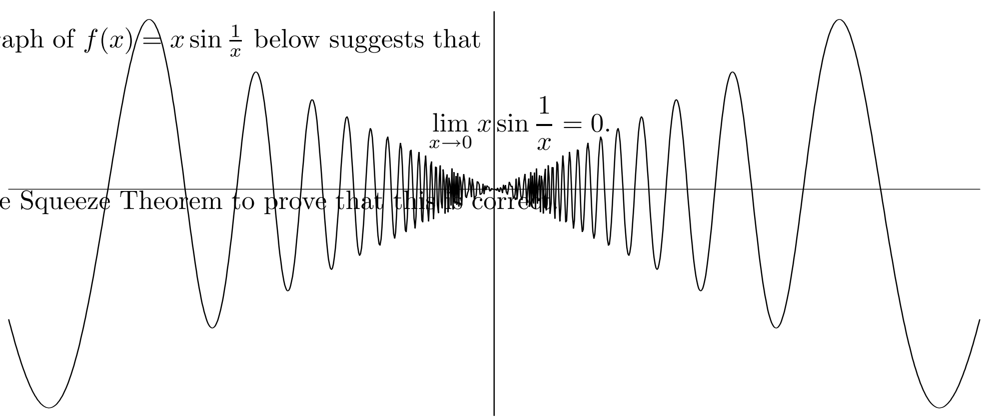
What is

$$\lim_{x\to b} f(x)?$$

3. The graph of $f(x) = x \sin \frac{1}{x}$ below suggests that

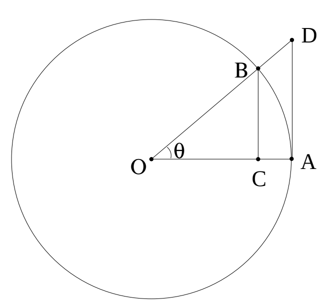
$$\lim_{x\to 0} x \sin \frac{1}{x} = 0.$$

Use the Squeeze Theorem to prove that this is correct.



4. In the unit circle below, use the area of triangle OBC, the area of triangle ODA, and the area of the sector of the circle OBA with the Squeeze Theorem to find the following limit:

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta}$$



5. Let $f(x)$ be a function that has x-intercepts at $x = 0$, $x = 2$, and $x = -2$; y-intercept $(0, 0)$; horizontal asymptote $y = -1$; and vertical asymptotes at $x = 3$ and $x = -3$:

   a) Graph $f(x)$. Is your graph unique? If so, why? If not, how many distinctive shapes can your graph have?

   b) As $x \to 3^+$ then $f(x) \to$____?

   c) As $x \to 3^-$ then $f(x) \to$____?

   d) As $x \to -3^+$ then $f(x) \to$____?

   e) As $x \to -3^-$ then $f(x) \to$____?

6. Given below is a *one-parameter family of functions.* That is, it is a collection of many functions f_t - one for each value of the parameter t. Determine which values of this parameter give a continuous function.

$$f_t(x) = \begin{cases} t \sin x, & x \leq \frac{\pi}{2}; \\ (1-t)x + t^2, & x > \frac{\pi}{2}. \end{cases}$$

   Graph f_t for every value of t for which f_t is continuous.

7. **More limits!** Compute

   a) $\displaystyle\lim_{x \to 1} \frac{1 - \frac{1}{x}}{1 - \frac{1}{\sqrt{x}}}$ b) $\displaystyle\lim_{x \to 1} \left(\frac{1}{x} - 1\right)\left(\frac{x}{x-1}\right)$ c) $\displaystyle\lim_{x \to 2} \frac{\sqrt{x+2} - 2}{x - 2}$