Emerging Scholars
University of Texas

Fall 1997

WORKSHEET 4

1. a) For each of the limits, graph the expression given, then use your graph to evaluate the limit.

$$\lim_{x\to -1^+} \frac{x^2+x}{|x^2+x|} \qquad\qquad \lim_{x\to 1}\,(1-|x-1|)$$

b) For each of the limits below, use your calculator to evaluate the expression for at least 4 different values of x. Use this data to guess the limit.

$$\lim_{x\to 0^+} \frac{x-\sin x}{x} \qquad \lim_{x\to 0} \frac{\sin^2(2x)}{x\sin x} \qquad \lim_{x\to 0} \frac{x}{\tan x}$$

Note: Four of these five problems have appeared on Dr. Davis's previous exams.

2. A function is said to be *continuous at the point* x_0 if

i. $f(x_0)$ is defined;
ii. $\lim_{x\to x_0} f(x)$ exists;
iii. and $\lim_{x\to x_0} f(x) = f(x_0)$.

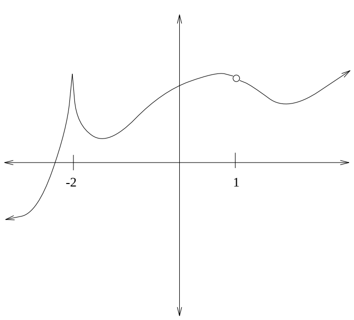
a) Sketch a graph of a discontinuous function for each of the following:
- condition i. holds, but condition ii. does not
- condition ii. holds, but condition i. does not
- conditions i. and ii. both hold, but condition iii. does not

b) Classify your examples as removable discontinuities, jump discontinuities, or asymptotes.

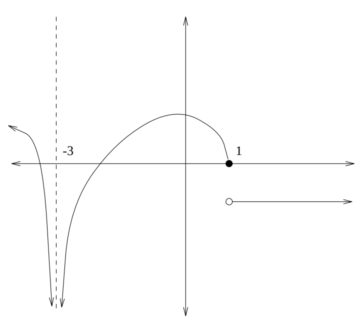
c) Could you have drawn examples which would have been classified differently?

3. Determine whether the following functions are continuous at the points given. At discontinuous points, indicate which of the conditions from Problem 2 do not hold.

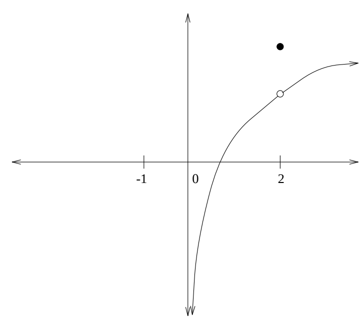
a) $x_0 = -2, 1$



b) $x_0 = -3, 1$



c) $x_0 = -1, 0, 2$



4. Graph the following functions:

$$f(x) = \begin{cases} x^2+5, & x < -1; \\ 6x, & x = -1; \\ 27x+33, & x > -1. \end{cases} \qquad g(x) = \begin{cases} \theta \sin\frac{1}{\theta}, & \theta \neq 0; \\ 0, & \theta = 0. \end{cases}$$

Determine whether $f(x)$ is continuous at $x_0 = -1$ and whether $g(x)$ is continuous at $x_0 = 0$. Fully justify your answers using the three parts of the definition of continuity given in Problem 2.

5. Suppose f, g, and h are functions and that

$$g(x) \leq f(x) \leq h(x) \qquad \forall x \in \mathbf{R}.$$

Furthermore, suppose that

$$\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L.$$

What is $\lim_{x\to a} f(x)$? Why?
Hint: Draw a picture.

6. Evaluate the following limits algebraically (you may need to be clever for some of them.)

a) $\displaystyle\lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{x-3}$ b) $\displaystyle\lim_{x\to -2} \frac{x^2+5x+6}{x^2+x-2}$ c) $\displaystyle\lim_{x\to 0} \frac{\sqrt{4+h}-2}{h}$ d) $\displaystyle\lim_{x\to 8} \frac{\sqrt[3]{x}-2}{x-8}$

e) $\displaystyle\lim_{x\to 3} \frac{1/x-1/3}{x-3}$ f) $\displaystyle\lim_{x\to 0} (1+x)^{1/x}$

7. Given below is a *one-parameter family of functions*. That is, it is a collection of many functions f_t - one for each value of the parameter t. Determine which values of this parameter give a continuous function.

$$f_t(x) = \begin{cases} t\sin x, & x \leq \frac{\pi}{2}; \\ (1-t)x + t^2, & x > \frac{\pi}{2}. \end{cases}$$

Graph f_t for every value of t for which f_t is continuous.