Emerging Scholars
University of Texas

Fall 1997

WORKSHEET 3

1. a) Graph the function $f(x) = \dfrac{1}{|x^2 - 2|}$.

   b) Give a piecewise definition for $f(x)$ which does not use the absolute value function.

2. Suppose an object is moving in a straight line according to the formula $s(t) = 5t^2 + 7$. Here the time, t, is measured in seconds and the distance traveled, s, is measured in meters.

   a) Find the average velocity over the time period from $t = 7$ to $t = 8$. If you use this as an estimate for the instantaneous velocity at $t = 7$, is it an over-estimate or under-estimate?

   b) Note that $8 = 7+1$. A reason for writing 8 as $7+1$ might be to stress the idea that we are interested in what is happening at time $t = 7$, so we'll write everything in terms of this. The **other** time we looked at in part a) was one second later than our time of interest, that is $t = 7 + 1$.

   Now suppose we are interested in looking at what happens during an **arbitrary** number of seconds (h seconds) past time $t = 7$. That is, we want to look starting at time $t = 7$ and ending at time $t = 7 + h$. Find the average velocity for the time period from $t = 7$ to $t = 7 + h$, where h is a constant.

   c) Simplify your expression from part b) so that there is no h in the denominator. Then plug in values $h = 1$, $h = 0.5$, and $h = 0.1$. What is the physical meaning of the numbers you computed?

   d) What happens to your expression as h becomes very close to zero? What is the meaning of considering such values of h?

   e) What is the instantaneous velocity of the object at time $t = 7$.

3. **Zeno's Tortoise.** You all know the story of the tortoise and the hare. There actually was a race many years ago between these two well known rivals! It was a 5-K run (the animals were smart enough to know the metric system is much better than the English system) and the tortoise was given a one kilometer head start. The hare could run 10 kilometers per hour compared to the tortoise's speed of one kilometer per hour. (This is actually a blazing speed for a tortoise, and you have to remember that he had won many preliminary races before finding himself pitted against the hare in this final round.) On the day of the race there happened to be a slug in the audience who was saying to anyone who would listen that the race was futile, and the hare had no chance of winning. His reasoning went like this:

   By the time the hare reaches the 1 km mark (where the tortoise starts), the tortoise will already have progressed 1/10 of a kilometer. Then, in the time it takes the hare to make up that distance, the tortoise will have progressed to a further location, maintaining his lead. Again, while the hare is running to this third spot, the tortoise will have moved still farther, leaving yet another gap. In this way, the tortoise will always remain in front.

   a) What is wrong with the slug's reasoning?

   b) At what point does the hare actually overtake the tortoise? (Sorry if you had your money on the tortoise, but he does indeed lose.)

4. a) Sketch a large graph of the function $f(x) = \sqrt[3]{x}$ over the interval $[-1, 3]$.

   b) Indicate on your graph the point where $x = 1$. What is the y-value there?

   c) Also indicate a point on the graph which is h units to the right of $x = 1$ along the x-axis. What is the y value of the graph at this point?

   d) What is the slope of the secant line connecting the points you have drawn? Evaluate this for $h = 2$, $h = 1$, and $h = 0.5$. Draw the secant lines on your graph which correspond to these values of h.

   e) Draw the tangent line at $x = 1$. Estimate the slope of this line.

5. Consider the following three functions:

$$f(x) = x^2 \qquad g(x) = \frac{1}{x^2} \qquad h(x) = \sqrt{x}.$$

For each of these, compute

a) $f(2)$

b) $f(x+2)$

c) $f(x+h)$

d) $\dfrac{f(x+h)-f(x)}{h}$ and simplify to the point where you have canceled the h in the denominator.

Hint for h(x): clear the radicals from the numerator by multiplying by a clever form of one.

6. A working light bulb is in a closed room with no windows. Outside the room, is a panel of three switches, one of which controls the light inside (up is on, down is off.) You may do anything you like to the three switches and then enter the room to inspect the light. After this, without any further experimentation, you must indicate which switch controls the light. What do you do?