WORKSHEET 37

1. Sketch the regions that give rise to the following integrals. Then rewrite it as a double integral with the integration order reversed.

a) $\displaystyle\int_0^1 \int_{\sqrt{x}}^{x^2} f(x,y)\,dy\,dx$

b) $\displaystyle\int_0^{2\pi} \int_0^{|\cos x|} f(x,y)\,dy\,dx$

c) $\displaystyle\int_a^b \int_{-a}^{-b} f(x,y)\,dx\,dy$

d) $\displaystyle\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} f(x,y)\,dy\,dx$

e) $\displaystyle\int_{2-\sqrt{3}}^{2+\sqrt{3}} \int_{1/y}^{4-y} f(x,y)\,dx\,dy$

f) $\displaystyle\int_1^2 \int_{\ln y}^{e^y} f(x,y)\,dx\,dy$

2. Evaluate:

a) $\displaystyle\int_0^{\pi} \int_y^{\pi} \frac{\sin x}{x}\,dx\,dy$

b) $\displaystyle\int_0^2 \int_x^2 e^{-y^2/2}\,dy\,dx$

c) $\displaystyle\int_0^{\ln 16} \int_{e^{x/2}}^4 \frac{dy\,dx}{\ln y}$

d) $\displaystyle\int_0^1 \int_{y^2}^1 \frac{e^x}{\sqrt{x}}\,dx\,dy$

3. a) Sketch the set of all points (x, y) in Euclidean coordinates on the plane that satisfy the inequalities

$$x_0 \le x \le x_0 + \Delta x \qquad \text{and} \qquad y_0 \le y \le y_0 + \Delta y.$$

b) Sketch the set of all points (r, θ) in polar coordinates on the plane that satisfy the inequalities

$$r_0 \le r \le r_0 + \Delta r \qquad \text{and} \qquad \theta_0 \le \theta \le \theta_0 + \Delta\theta.$$

c) Find the area of the ring between two circles, one of radius r_0, the other of radius $r_0 + \Delta r$.

d) What fraction of the area in c) is included between two rays whose angles differ by $\Delta\theta$?

e) Show that the area of the region in part b) is precisely

$$\left(r_0 + \frac{\Delta r}{2}\right) \Delta r \Delta\theta.$$

Mike Oehrtman

4. Sketch the regions that give rise to the following integrals:

a) $\displaystyle\int_0^{\pi}\int_0^{2a\sin\theta} r\,dr\,d\theta$ b) $\displaystyle\int_{-\pi/4}^{\pi/4}\int_1^{2} r\,dr\,d\theta$ c) $\displaystyle\int_0^{\pi/4}\int_0^{b\sec\theta} r\,dr\,d\theta$

5. Carry out the integrals in Problem 4.

6. a) Find the volume of the solid in the first octant bounded by the cylinder $x^2+y^2=4$ and the parabaloid $z=x^2+y^2$.

   b) Find the volume of the solid bounded above by $z=1-x^2-y^2$ and below by the xy-plane

7. Integrate the function f over the region Ω by changing to polar coordinates:

   a) $f(x,y)=\dfrac{1}{4a}(x^2+y^2)$
   $\Omega =$ disk of radius a centered at $(0,a)$

   b) $f(x,y)=y$
   $\Omega =$ region between the circles of radii a and b centered at the origin $(a<b)$