WORKSHEET 36

1. True or False. If false, make the statement true.

   a) If f is a differentiable function of x and y, then the directional derivative of f in the direction of the unit vector $\mathbf{u}$ is $D_{\mathbf{u}}f(x,y) = \nabla f(x,y) \cdot \mathbf{u}$.

   b) $\mathbf{a} \cdot \mathbf{b} = ||\mathbf{a}||\,||\mathbf{b}|| \sin\theta$.

   c) The minimum value of $\mathbf{a} \cdot \mathbf{b}$ is $||\mathbf{a}||$.

   d) The maximum value of $D_{\mathbf{u}}f(x,y)$ is π for any $f(x,y)$.

   e) $D_{\mathbf{u}}f(x,y) = 0$ for $\mathbf{u} = \frac{\nabla f(x,y)}{||\nabla f(x,y)||}$.

   f) The direction of **maximum** increase of f is given by $\nabla f(x,y)$.

2. The temperature distribution around a certain point (set here to be the origin $(0,0,0)$ in 3-space) is given by $T(x,y,z) = \sqrt{x^2+y^2+z^2}$. That is, the temperature at any point is equal to the distance from that point to the origin.

   a) Find ∇T.

   b) At the point $(1,2,-2)$, what direction would you travel to experience the greatest increase in temperature? the least?

   c) From the point $(1,2,-2)$, indicate two directions in which the rate of change in temperature would be zero. (Make these two directions which are NOT simply 180° apart.)

   d) Describe the level set consisting of all points (x,y,z) where the temperature is 3 units, that is, where $T(x,y,z) = 3$.

   e) Find an equation for the plane which is tangent to the surface you describe in part d) at the point $(1,2,-2)$.

   f) A particle traces the path $\mathbf{x}(t) = (\cos t, \sin t, t)$ for $0 \le t \le 2\pi$. At what point of the path is the lowest temperature experienced?

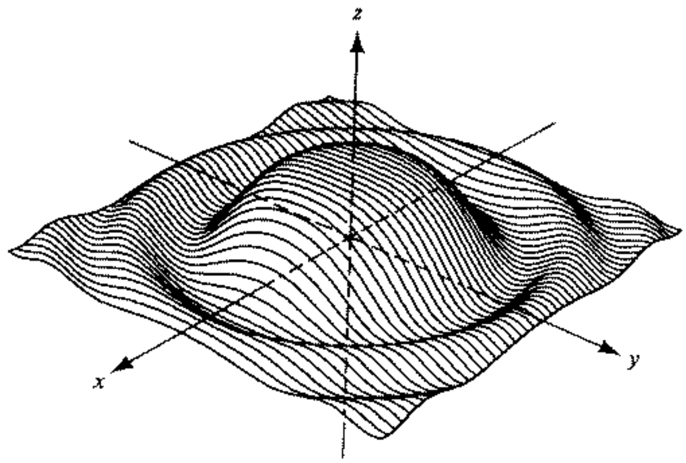
3. Let $f(x,y,z) = x^2 e^{-yz}$. Compute the rate of change of f in the direction of the unit vector

$$\mathbf{v} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \quad \text{at} \quad (1,0,0).$$

Mike Oehrtman

4. Assume that $\nabla f(\mathbf{x}) \neq \mathbf{0}$. Prove that $\nabla f(\mathbf{x})$ points in the direction along which f is increasing the fastest.
   (Hint: Think directional derivative.)

5. Below is the graph of a function $f\colon \mathbb{R}^2 \to \mathbb{R}$. Sketch ∇f as a vector field on $\mathbb{R}^2$.



6. Sketch the level set $f(x, y, z) = 1$ for the function f given in Problem 3. Find two nonparallel vectors at the point $(-1, 0, 0)$ that are tangent to this level set.

   Hint: to sketch the level surface, first consider what the intersections of this surface with the horizontal planes $z =$ constant look like.