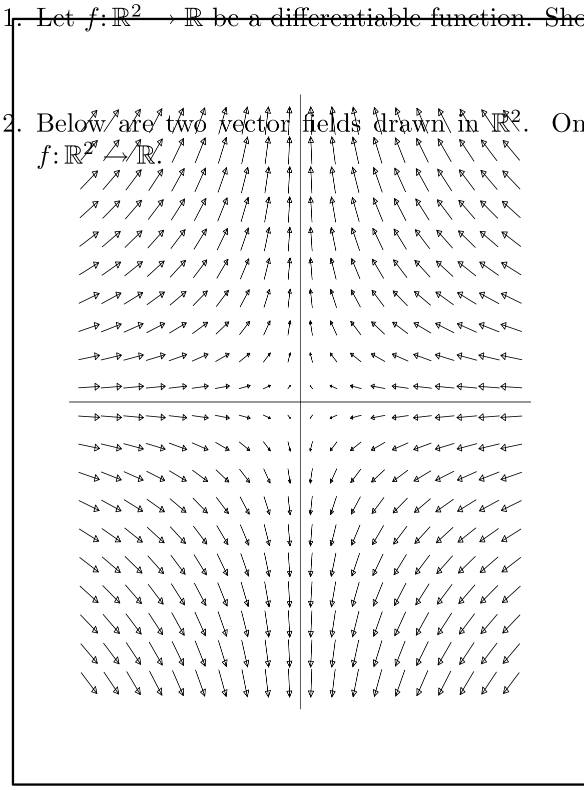
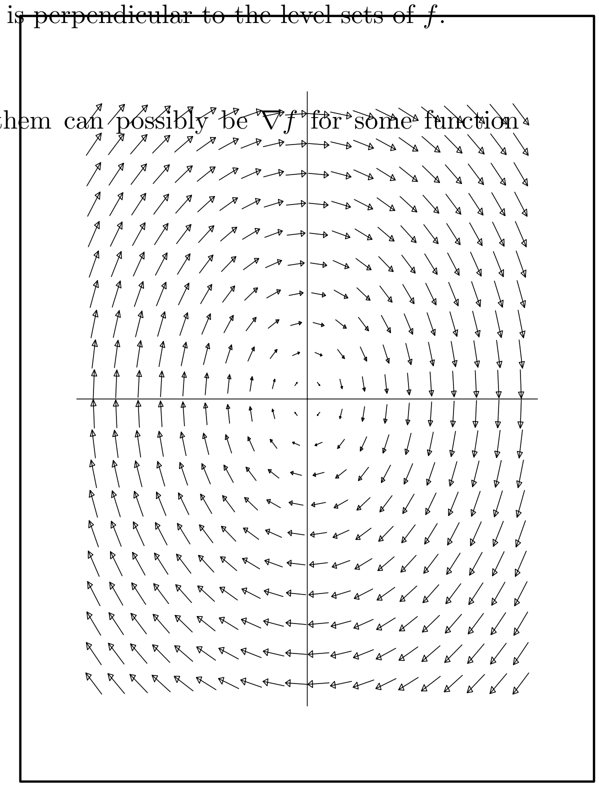
WORKSHEET 35

1. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function. Show that ∇f is perpendicular to the level sets of f.

2. Below are two vector fields drawn in $\mathbb{R}^2$. Only one of them can possibly be ∇f for some function $f: \mathbb{R}^2 \to \mathbb{R}$.





   a) Which of the vector fields is ∇f for some function $f: \mathbb{R}^2 \to \mathbb{R}$? Why?

   b) For the vector field you identified in part a), draw the level sets of f.

   c) Sketch the graph of f.

   d) Now draw a few solution curves to the differential equations represented by the vector field. That is, draw lines which are everywhere tangent to the vector field. If you followed a path on the graph of f directly above one of these curves, describe what you would be doing.

3. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x, y) = x^2 + y^2$.

   a) Find ∇f and sketch it as a vector field on the plane.

   b) Sketch a graph of f.

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4. A volcano just erupted and lava is streaming down the mountaintop. Suppose that the altitude of the mountain is given by

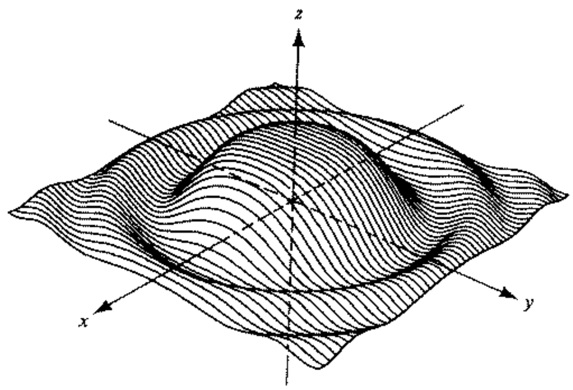
$$z(x, y) = he^{-(x^2+y^2)}$$

where h is the maximum height, and suppose also that lava flows in the direction of steepest descent.

   a) Find the projection on the xy-plane of the direction in which the lava flows away from the point $(1, 2, he^{-9})$.

   b) Find the projection on the xy-plane of the path taken by the lava as it flows down the mountaintop.

5. Below is the graph of a function $f\colon \mathbb{R}^2 \to \mathbb{R}$. Sketch ∇f as a vector field on $\mathbb{R}^2$.



6. Suppose that a duck is swimming in the circle $x = \cos t$, $y = \sin t$ and that the water temperature is given by the formula $T = x^2e^y - xy^3$. Find dT/dt, the rate of change in temperature the duck might feel:

   a) by the chain rule;

   b) by expressing T in terms of t and differentiating.