WORKSHEET 34
a.k.a. Practice Exam 2

1. Show that $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x}$ for $z = 2^{x^3 y}$.

2. Find the plane perpendicular to the path $(t^2, t, \cos 2\pi t)$ at time $t = 1/4$.

3. An object is traveling along the path $(a \sin t, b \cos t, t)$ for $0 \leq t < 2\pi$ where a and b are constants with $a > b$. At what location(s) is the object moving the fastest?

4. Consider the paths
$$\gamma(t) = (\cos t, t, t^2)$$
$$\text{and} \qquad \sigma(t) = (t^2 + 1, -t, \sin t).$$
Find the equation of a plane which is tangent to both of these curves at a point of their intersection.

5. a) Sketch a graph in polar coordinates of $r = 1 - 2\sin\theta$.
   b) Find the area of the smallest loop.
   c) Set up the integral which gives the arclength around this loop.

6. Let A, B, and C be the vertices of a triangle. Let $\mathbf{A} = \vec{OA}$, $\mathbf{B} = \vec{OB}$, and $\mathbf{C} = \vec{OC}$. Let P be the point that is on the line segment joining A to the midpoint of the edge BC and twice as far from A as from the midpoint. Show that $\vec{OP} = \mathbf{A} + \mathbf{B} + \mathbf{C})/3$.

7. Given that the orbit of a planet about a sun (located at the origin) is
$$r = \frac{k}{1 + e\cos\theta}$$
for some constants $k > 1$ and $0 < e < 1$, find the points where the position vector is perpendicular to the velocity vector. At what point is the planet farthest from the sun?

8. a) For a parameterized path $\mathbf{r}$ with velocity vector $\mathbf{v}$ and acceleration vector $\mathbf{a}$, simplify the expression
$$[(\mathbf{r} \times \mathbf{v}) \cdot \mathbf{a}]'.$$
   b) If $\mathbf{r}$ is the path of a satellite about the earth at the origin, find the exact value of $(\mathbf{r} \times \mathbf{v}) \cdot \mathbf{a}$.

Mike Oehrtman