

WORKSHEET 29

1. An object's position at time t is given by the path $\sigma(t)$. Suppose that $\sigma'(2) = (1, -3, 7)$, $\sigma'(2.5) = (2, 0, 4)$, $\sigma''(2) = (2, 3, 1)$, and $\sigma''(2.5) = (0, 1, -1)$.
 - a) At time $t = 2$, is the object's velocity changing?
 - b) At time $t = 2$, is the object's speed changing? If so, is it increasing or decreasing?
 - c) At time $t = 2.5$, is the object's velocity changing?
 - d) At time $t = 2.5$, is the object's speed changing? If so, is it increasing or decreasing?
2. Suppose that an object's path is given by $\gamma(t) = (t^2, \sin t)$.
 - a) Sketch the path from time $t = -\pi$ to time $t = \pi$.
 - b) Compute the velocity vector at times $t = 0$ and times $t = 1$ and draw them on your graph based at the object's position at each of those times.
 - c) Set up an integral that indicates the distance traveled by the object from time $t = 0$ to time $t = 1$.
 - d) Give a parameterization of this path in polar coordinates.
3. Recall from Worksheet 27, the sad predicament that our good friends, the Idiots, faced:

In the Odd Galaxy, there is a planet called Id. The Idiots, the inhabitants of Id, have noticed that their planet goes in a large slow circular orbit given by the path $\mathbf{I}(t) = (\sin t, \cos t, 0)$. The unit of time is one million Earth years, and the unit of length is one Idian Astronomical Unit (IAU). Their astronomers notice a very large asteroid with path given by $\mathbf{R}(t) = (\csc t, 0, \cot t)$ for $0 < t < \pi$.

- a) At what angle will the trajectories of the planet and the asteroid collide?
- b) What will the relative **speed** of the asteroid (with respect to the planet Id) be when the two bodies collide?
- c) The current Idian date is $t = \pi/4$. At Idian date $t = \pi/6$ (notice that is over a quarter of a million Earth years ago!) one of the greatest prophetic Idiots of all time predicted that a giant ball of flame would fall from the sky and destroy the planet. How far from planet Id was the asteroid at the time of this prediction? How far away is it now?
- d) How far has the asteroid traveled since the prophecy?

- e) “Hurling” is the most popular Idiotic sport. It involves hurling a small ball at another player who then tries to hit it with a stick. One tactic used by only the most talented “hurlers” is to put spin on the ball so that it curves and is difficult to hit. Many idiots believe that the asteroid is a giant “hurlball” and they are greatly impressed with the abilities of the cosmic “hurler”. Find the greatest curvature of the asteroid’s path. What will the curvature be at the time of collision. What religious significance does this hold?
- f) Find the acceleration of the asteroid at time t . When is the asteroid speeding up? When is it slowing down? What is the rate of change of the speed of the asteroid at time $t = \pi/4$?
- g) Give a parameterization of the path of the planet in polar coordinates. (You may restrict your attention to the (x, y) -plane.)