WORKSHEET 27

1. a) In polar coordinates, the orbit of Mercury is given by the equation

$$r = \frac{3.442 \times 10^7}{1 - 0.206 \cos\theta}$$

where r is the distance in miles from the sun. Find the distance of Mercury's closest and farthest approach to the sun.

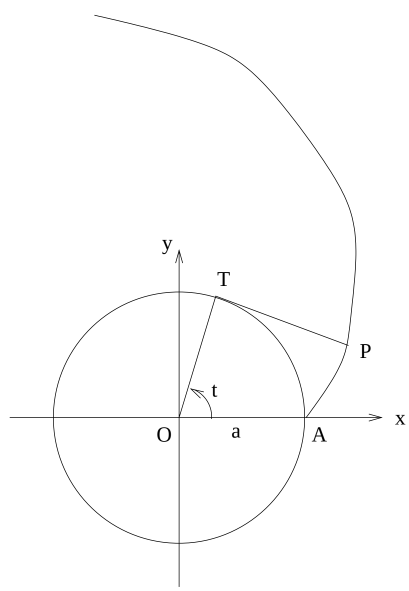
b) The orbit of Saturn is given by the equation

$$r = \frac{1.4228}{1 - 0.0543 \cos\theta}$$

where r is the distance in billions of kilometers from the sun. Find the disance of Saturn's closest and farthest approach to the sun.

c) Sketch the orbits of Mercury and Saturn.

2. **Involute of a circle.** If a string wound around a fixed circle is unwound while held taut in the plane of the circle, its end traces an *involute* of the circle. Let the fixed circle be located with its center at the origin O and have radius a. Let the initial position of the tracing point P be $A = (a, 0)$ and let the unwound portion of the string PT be tangent to the circle at T. Derive parametric equations for the involute, using angle AOT as the parameter t.



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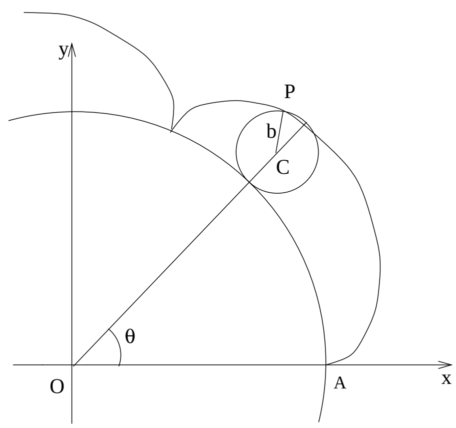
3. In the Odd Galaxy, there is a planet called Id. The Idiots, the inhabitants of Id, have noticed that their planet goes in a large slow circular orbit given by the path $\mathbf{I}(t) = (\sin t, \cos t, 0)$. The unit of time is one million earth years, and the unit of length is one Idian Astronomical Unit (IAU). Their astronomers notice a very large asteroid with path given by $\mathbf{R}(t) = (\csc t, 0, \cot t)$ for $0 < t < \pi$. Are the Idiots in danger?

4. **Epicycloids.** When a circle rolls externally along the circumference of a second, fixed circle, any point P on the circumference of the rolling circle describes an *epicycloid.* Let the fixed circle have its center at the origin O and have radius a. Let the radius of the rolling circle be b and let the initial position of the tracing point be $A = (a, 0)$. Determine parametric equations for the epicycloid, using as parameter the angle θ from the positive x-axis to the line of centers.



5. **Hypocycloids.** When a circle rolls on the inside of a fixed circle, any point P on the circumference of the rolling circle describes a *hypocycloid.* Let the fixed circle be $x^2 + y^2 = a^2$, let the radius of the rolling circle be b, and let the initial position off the tracing point P be $A = (a, 0)$. Use the angle θ from the positive x-axis to the line of centers as parameter and determine parametric equations for the hypocycloid.