WORKSHEET 26

1. a) Given the following polar coordinates for a point in $\mathbb{R}^2$, determine Euclidean coordinates that represent the same point. Is there more than one such representation?

$$\left(3, \frac{\pi}{6}\right) \qquad (0, \pi^2) \qquad \left(-\frac{1}{4}, \frac{\pi}{4}\right) \qquad (-8, -37\pi)$$

   b) Given the following Euclidean coordinates for a point in $\mathbb{R}^2$, determine polar coordinates that represent the same point. Is there more than one such representation?

$$(1, 0) \qquad (0, 1) \qquad (-2\sqrt{3}, 2) \qquad (-4, -3)$$

2. For a given point in $\mathbb{R}^2$ with polar coordinates (r, θ) list all pairs (r', θ') which determine the same point.

3. Find the intersections of each of the following pairs of graphs in polar coordinates. Use both graphical and algebraic methods for each pair.

$$r = \sin^2\theta \qquad r = \theta \qquad \theta = \frac{3\pi}{2}$$

$$r = -1 \qquad r = \pi - \theta \qquad r = \theta \sin\theta$$

4. a) Explain why the area enclosed between the polar graph of a function $r = f(\theta)$ and the two lines $\theta = \theta_a$ and $\theta = \theta_b$ is NOT equal to $\int_{\theta_a}^{\theta_b} f(\theta)\, d\theta$.

   b) If you partition the interval $[\theta_a, \theta_b]$ how could you estimate the area enclosed between the angles of one subinterval $[\theta_i, \theta_{i+1}]$ and the graph of $r = f(\theta)$? Write down a formula.
   (Hint: Use a wedge shaped piece of a circle (of what radius?) to approximate the region over the subinterval $[\theta_i, \theta_{i+1}]$. What percent of the area of the whole circle does this represent?)

   c) Write down the exact area enclosed as a limit of Riemann sums.

5. Sketch the graphs of $r = \cos 2\theta$ and $r = \frac{1}{2} + \frac{1}{2}\cos 4\theta$. Find the area bounded by each of these graphs.

7. Let U be a bounded region of the plane. Suppose that you can determine the length of any cross section of this region through the origin. Can you determine the area of U? If so how? If not, give an example of two regions with the same lengths of cross sections through the origin, but which have different areas.

Mike Oehrtman