WORKSHEET 24

1. Find the line in the plane of $(0,0,0)$, $(2,2,0)$, and $(0,1,-2)$ that passes through the origin perpendicular to the line
$$\frac{x}{3} = \frac{y}{2} = 2z.$$

2. Two surface ships on manuevers are trying to determine a submarine's course and speed to prepare for an aircraft intercept. Ship A is located at $(4,0,0)$ while ship B is located at $(0,5,0)$. All coordinates are given in thousands of feet. Ship A locates the submarine in the direction of the vector $2\mathbf{i} + 3\mathbf{j} - (1/3)\mathbf{k}$, and ship B locates it in the direction of the vector $18\mathbf{i} - 6\mathbf{j} - \mathbf{k}$. Four minutes ago, the ship was located at $(2,-1,-1/3)$. The aircraft is due in 20 minutes. Assuming the submarine moves in a straight line at a constant speed, to what position should the surface ships direct the aircraft?

3. Two helicopters, H_1 and H_2, are traveling together. At time $t = 0$ hours, they separate and follow different straight line paths given by
$$\begin{aligned} H_1: &\quad x = 6 + 40t, \quad y = -3 + 10t, \quad z = -3 + 2t \\ H_2: &\quad x = 6 + 110t, \quad y = -3 + 4t, \quad z = -3 + t \end{aligned}$$
all coordinates measured in miles. Due to system malfunctions, H_2 stops its flight at $(446,13,1)$ and, in a negligible amount of time, lands at $(446,13,0)$. Two hours later, H_1 is advised of this fact and heads toward H_2 at 150 mph. How long will it take H_1 to reach H_2?

4. Prove that four points A, B, C, and D are coplanar (lie in a common plane) if and only if $\vec{AD} \cdot (\vec{AB} \times \vec{BC}) = 0$.

5. If the four vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are coplanar, show that $(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = \mathbf{0}$.

6. a) Let $\mathbf{F}: \mathbb{R} \to \mathbb{R}^3$. Describe the domain and range of this function. What uses might such a function have?

   b) If $\mathbf{F}(t) = (x(t), y(t), z(t))$, then the derivative $d\mathbf{F}/dt$ is defined by
$$\frac{d\mathbf{F}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right).$$
   Find $d\mathbf{F}/ds$ as a function of s if $t = s^2 - 1$ and $\mathbf{F}(t) = (1, \sin(t+1), \sqrt{t})$.

   c) Prove the following differentiation rules for two vector functions $\mathbf{U}$ and $\mathbf{V}$,
$$\frac{d}{dt}(\mathbf{U} \cdot \mathbf{V}) = \frac{d\mathbf{U}}{dt} \cdot \mathbf{V} + U \cdot \frac{d\mathbf{V}}{dt}$$
$$\frac{d}{dt}(\mathbf{U} \times \mathbf{V}) = \frac{d\mathbf{U}}{dt} \times \mathbf{V} + U \times \frac{d\mathbf{V}}{dt}.$$

Mike Oehrtman