

WORKSHEET 22

1. Show that the point $(\frac{1}{\sqrt{2}}, \frac{-1}{2}, \frac{1}{2})$ lies on the unit sphere. What is the equation of the plane tangent to the unit sphere at this point?
2. **“The sexiest binary operation on vectors in three-space:”**
Recall the *cross product* of two vectors $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ in \mathbb{R}^3 is the determinant of the matrix

$$\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{pmatrix}.$$

List the properties of the cross product. Which of these can you justify?

3. Let \mathbf{u} be a unit vector, and \mathbf{B} be any vector which is not parallel to \mathbf{u} . Describe what happens to the sequence of vectors

$$\mathbf{u} \times \mathbf{B}, \quad \mathbf{u} \times (\mathbf{u} \times \mathbf{B}), \quad \mathbf{u} \times (\mathbf{u} \times (\mathbf{u} \times \mathbf{B})), \quad \dots$$

4. How many planes contain at least three of the points $(1, 0, 2)$, $(-1, 3, 1)$, $(4, 4, 4)$, and $(0, 0, 0)$? Find the distance from each point to any of the planes not containing that point.
5. A fluid flows across a plane surface with uniform vector velocity \mathbf{v} . Let \mathbf{n} be a unit normal to the plane surface. How much fluid passes through a unit area of the plane in unit time?
6. a) Suppose that $\mathbf{a} \cdot \mathbf{b} = \mathbf{a}' \cdot \mathbf{b}$ for all \mathbf{b} . Show that $\mathbf{a} = \mathbf{a}'$.
b) If $\mathbf{a} \times \mathbf{b} = \mathbf{a}' \times \mathbf{b}$ for all \mathbf{b} , is it necessary that $\mathbf{a} = \mathbf{a}'$?
7. Consider the parallelepiped spanned by three vectors \mathbf{a}, \mathbf{b} and \mathbf{c} in \mathbb{R}^3 .
a) Prove that the volume of the parallelepiped is the *scalar triple product*

$$|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|.$$

- b) Find the volume of the parallelepiped spanned by the vectors

$$(1, 0, 1), \quad (1, 1, 1), \quad \text{and} \quad (-3, 2, 0).$$