WORKSHEET 21

1. a) Draw two vectors **u** and **v** based at the same point and which are neither parallel nor orthogonal. Label the angle between the vectors θ, and add to your picture the two vectors $\mathbf{proj_u v}$ and $\mathbf{proj_v u}$.

   b) If you replace **v** with any vector pointing in the same direction, will $\mathbf{proj_v u}$ change? Will $\mathbf{proj_u v}$?

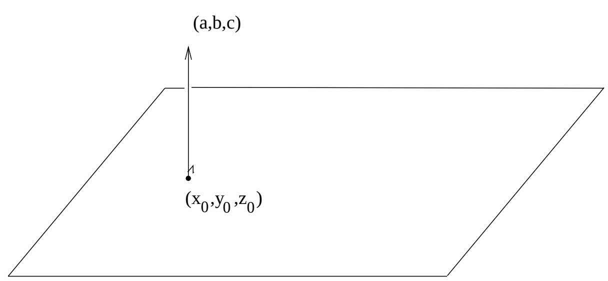
   c) If you replace **u** with any vector pointing in the same direction, will $\mathbf{proj_v u}$ change? Will $\mathbf{proj_u v}$?

   d) Write a formula for the LENGTHS of $\mathbf{proj_u v}$ and $\mathbf{proj_v u}$. Do these formulas depend on the length of **u** or the length of **v**? How does this match up with your answers to parts b) and c)?

   e) Write a formula for the VECTORS $\mathbf{proj_u v}$ and $\mathbf{proj_v u}$.

2. Describe the geometric meaning of the dot product.

3. Below is a picture of a plane passing through the point (x_0, y_0, z_0) and perpendicular to the vector (a, b, c). Let (x, y, z) be any point in $\mathbb{R}^3$.



   a) Write out the coordinates of the vector which is based at the point (x_0, y_0, z_0) and ends at the point (x, y, z).

   b) If the point (x, y, z) lies in the plane, what can be said about the relationship of the vector found in part a) with the vector (a, b, c).

   c) What is the equation of the plane?

   d) Find a normal vector to the plane $4x - 7y - z - 9 = 0$, and find a point on the plane.

   e) Plug the normal vector and the point found in part d) into your equation from part c). Does this give the same plane as $4x - 7y - z - 9 = 0$?

Mike Oehrtman

4. Now suppose from the situation in Problem 3 that the point (x, y, z) is not on the plane.

   a) Redraw the picture, then drop a perpendicular from the point (x, y, z) to the plane. Also draw and label the vector based at the point (x_0, y_0, z_0) and ending at the point (x, y, z). What are the coordinates of this vector?

   b) The distance from the point (x, y, z) to the plane is now given by the length of the projection of some vector onto another. Identify this projection.

   c) What is the distance from the point (x, y, z) to the plane?

5. a) Write down the equation of a plane which is parallel to the plane given by

$$-\pi^2 x + 4ey - \sqrt{7}z + 3 = 0$$

   and which contains the origin.

   b) How far apart are these two planes?

   c) How far along the x-axis would you have to translate one of these planes to make it coincide with the other? What if you translate along the y-axis? The z-axis?

6. Show that the point $(\frac{1}{\sqrt{2}}, \frac{-1}{2}, \frac{1}{2})$ lies on the unit sphere. What is the equation of the plane tangent to the unit sphere at this point?

7. Find a vector orthogonal to the indicated figure in the indicated dimension:

   a) The line $x + 2y + 3 = 0$ in two dimensions.

   b) The plane $x + 2y + 3z + 4 = 0$ in three dimensions.

   c) The hyperplane $x + 2y + 3z + 4w + 5 = 0$ in four dimensions.

8. Find the distance from the indicated point to the indicated figure in the appropriate dimension. Use the last problem, and don't just plug in to some formula you don't understand!

   a) $P = (3, 4)$ and the line $x + 2y + 3 = 0$.

   b) $P = (1, 3, 4)$ and the plane $x + 2y + 3z + 4 = 0$.

   c) $P = (1, 2, 3, 4)$ and the hyperplane $x + 2y + 3z + 4w + 5 = 0$.