WORKSHEET 20

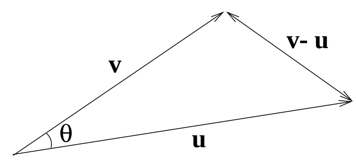
1. Recall that if **u** and **v** are vectors in $\mathbb{R}^n$, we define their *dot product* to be

$$\mathbf{u} \cdot \mathbf{v} = ||\mathbf{u}|| \, ||\mathbf{v}|| \cos\theta$$

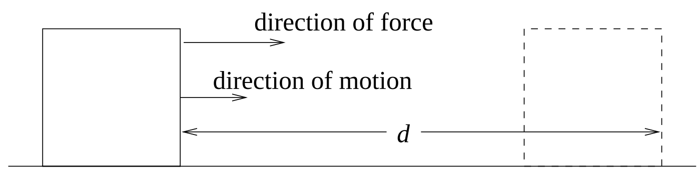
where θ measures the smaller angle made by **u** and **v** when their initial points coincide. For vectors $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ in $\mathbb{R}^3$, prove the formula

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3.$$

Hint: Consider the triangle formed by the three vectors **u**, **v**, and $\mathbf{v} - \mathbf{u}$. Write down the lengths of these three vectors and then apply the law of cosines.

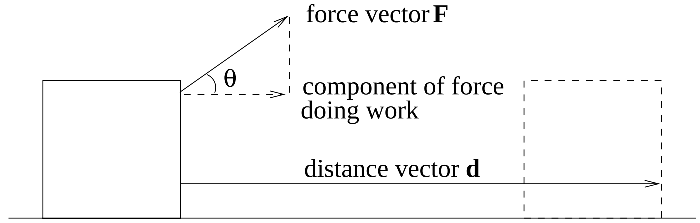


2. You may be familiar with the equation $W = fd$ from physics. It says that the amount of work W done in moving an object a distance d by applying a constant force f in the direction of motion is the product of f and d.



a) Suppose that the force applied to the object is not constant but varies as the object moves. How could you modify this formula to compute the work done? Justify your answer. What assumptions did you make?

b) Suppose a constant force is applied to the object but now in a direction different from the direction of motion. For example, a rope may pull a heavy block at an upward angle but the block slides horizontally. Represent the force by a vector **F** which points in the direction of the force and whose magnitude is the amount of force being applied. Represent the distance with a vector **d** pointing in the direction of motion and with magnitude equal to the distance traveled. Find a formula for the work done in this case in terms of the vectors **F** and **d**.



Mike Oehrtman

c) How much work does it take to slide a crate 20 m along a loading dock by pulling on it with a constant force of 200 Newtons at an angle of 30° from the horizontal?

d) What general approach would you follow to come up with a formula for the work done if the force vector was continuously changing and the path of motion was not a straight line.

3. Find the angles between the following two curves at each intersection point:

$$y = x^3 \qquad \text{and} \qquad y = \sqrt{x}.$$

4. What does it mean for vectors to be orthogonal? Explain how you can determine if two vectors are orthogonal, and why that method works. What does it mean for vectors to be parallel? Explain how you can determine if two vectors are parallel, and why it is that your method works.

5. Find conditions on the vectors $\mathbf{u}$ and $\mathbf{v}$ which ensure that $\mathbf{u}+\mathbf{v}$ is orthogonal to $\mathbf{u}-\mathbf{v}$.

6. A 1-kilogram (1-kg) mass located at the origin is suspended by ropes attached to the points $(1, 1, 1)$ and $(-1, -1, 1)$. If the force of gravity is pointing in the direction of the vector $(0, 0, -1)$, what is the vector describing the force along each rope?

   [Hint: Use the symmetry of the problem. A 1-kg mass weighs 9.8 Newtons (N).]