WORKSHEET 19

1. A ship is at position $(1,0)$ on a nautical chart (with north the positive y direction sights a rock at position $(2,4)$. The ship is pointing due north and traveling at a speed of 4 knots relative to the water. There is a current flowing due east at 1 knot; The units on the chart are nautical mile; 1 knot = 1 nautical mile per hour.

   a) If there were no current, what vector **u** would represent the velocity of the ship relative to the sea bottom?

   b) If the ship were just drifting with the current, what vector **v** would represent its velocity relative to the sea bottom?

   c) What vector **w** represents the total velocity of the ship?

   d) Where would the ship be after 1 hour?

   e) Should the captain change course?

   f) What if the rock were an iceberg?

2. In 1618, Johann Kepler discovered that the square of the time required by a planet for one revolution around the sun is proportional to the cube of its mean distance from the sun. In fact, this law applies to all objects in orbit about a much larger object. On April 24, 1990 NASA launched the Hubble Space Telescope which now orbits the Earth 17 times a day at a distance of 3732 miles.

   a) How far away is the moon?

   b) A *geosynchronous* orbit is one in which the sattelite remains over the same geographical location on the Earth. What is the radius of a geosynchronous orbit?

Mike Oehrtman

3. Let $\mathbf{u}$ be the vector from the origin to (u_1, u_2, u_3). Let $\mathbf{v}$ be the vector from the origin to (v_1, v_2, v_3). Let $\mathbf{w}$ be the vector from the origin to (w_1, w_2, w_3). Let a and b denote real numbers.

   a) For each of the following statements, say if they are true always, sometimes, never, or are meaningless. If always, never, or meaningless, explain why. If sometimes give examples of both situations.

      i) $(a\mathbf{u}) \cdot (b\mathbf{v}) = (ab)\mathbf{u} \cdot \mathbf{v}$
      ii) $(a\mathbf{u}) \cdot (b\mathbf{v}) = (ab)\mathbf{u}\mathbf{v}$
      iii) $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$
      iv) $a \cdot (\mathbf{u} + \mathbf{v}) = a \cdot \mathbf{u} + a \cdot \mathbf{v}$
      v) $\mathbf{w}(\mathbf{u} + \mathbf{v}) = \mathbf{w}\mathbf{u} + \mathbf{w}\mathbf{v}$
      vi) $\mathbf{w} \cdot (\mathbf{u} + \mathbf{v}) = \mathbf{w} \cdot \mathbf{u} + \mathbf{w} \cdot \mathbf{v}$
      vii) $\mathbf{w} \cdot (\mathbf{u} \cdot \mathbf{v}) = (\mathbf{w} \cdot \mathbf{u}) \cdot \mathbf{v}$
      viii) $(\mathbf{u} \cdot \mathbf{v})\mathbf{w} = (\mathbf{w} \cdot \mathbf{u})\mathbf{v}$
      ix) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

   b) Verify that $(a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}$.

4. Prove that if $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is orthogonal to $c\mathbf{v} + d\mathbf{w}$ for any scalars c and d. What conclusions can you make?