WORKSHEET 15

1. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $\sum_{n=0}^{\infty} a_n 9^n$ converges and that $\sum_{n=0}^{\infty}(-1)^n a_n 12^n$ diverges. What if anything can be said about

a) the convergence of $\displaystyle\sum_{n=0}^{\infty} a_n 7^n$

b) the absolute convergence of $\displaystyle\sum_{n=0}^{\infty} a_n (-7)^n$

c) the absolute convergence of $\displaystyle\sum_{n=0}^{\infty} a_n 9^n$

d) the convergence of $\displaystyle\sum_{n=0}^{\infty} (-1)^n a_n 9^n$

e) the convergence of $\displaystyle\sum_{n=0}^{\infty} a_n 10^n$

f) the convergence of $\displaystyle\sum_{n=0}^{\infty} a_n (-15)^n$

g) the convergence of $\displaystyle\sum_{n=0}^{\infty} a_n 15^n$

h) the convergence of $\displaystyle\sum_{n=500}^{\infty} a_n 15^n$

2. Sketch and label on a number line the regions where you know the series in Problem 1 converges and where it diverges. What can be said about

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}?$$

3. Let $f(x)$ be a *smooth* function, that is, the derivatives $f^{(n)}(x)$ exists for all $n = 1, 2, 3, \ldots$ The *Taylor series for* $f(x)$ *about* $x = a$ is defined to be

$$T_a(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Find the Taylor series about $x = a$ for the following:

a) $f(x) = x^2$ b) $f(x) = x$ c) $f(x) = 2^x$

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4. Consider the function $f(x) = e^{-1/x^2}$.

   a) Graph $f(x)$ **without the aid of a calculator**.

   b) Notice that $f(0)$ is not defined. (You did indicate that in your graph didn't you?) Define $f(0)$ to be a value so that $f(x)$ is continuous at $x = 0$.

      You may either take my word for it or prove that $f(x)$ is now infinitely differentiable at $x = 0$. That is, not only have you made $f(x)$ continuous at $x = 0$, but in fact, all of its derivatives $f^{(n)}(x)$ exist there.

   c) Find the Taylor series for $f(x)$ centered at $x = 0$. Does this equal $f(x)$?

5. Important in the theory of probability is the *normal density function*

$$n(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}.$$

   a) Graph $n(x)$.

   b) Various probabilites can be computed as the area under certain sections of this curve. Typically, for various values of x, we may want to compute

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}y^2}\, dy$$

      which is called the *normal distribution function.* What makes computing this integral difficult?

   c) Find the Taylor series centered at $x = 0$ for the normal distribution $N(x)$. Why might this be useful?