

WORKSHEET 5**1. MacGyver Mathematics**

- a) For the following task, you are equipped with a hacksaw and a solar panel which is hooked up to a meter. By laying flat objects on the panel and reading the meter, you can determine how much surface area the object spans on the panel. Suppose you have a solid object that you don't mind destroying. Describe a way to accurately estimate its volume.

What could you do to make your estimate even more accurate?

- b) Say that you want to find the area of a certain cross-sectional plane of a rock. One way to find it is by sawing the rock in two and measuring the area directly. But suppose you do not want to ruin the rock. However, you do have a very accurate measuring glass which gives you excellent volume measurements. How could you use the glass to get a good estimate of the cross-sectional area?

What mathematical principle are you employing here?

2. Derive the volumes of the following solids using calculus:

- a) a ball of radius r ;
b) a right circular cone of radius r and height h ;
c) a right circular cylinder of height h and radius r .

3. a) A deck of cards is stacked straight and then pushed to the side so that the stack becomes skewed. In which of these two positions does the deck occupy more volume?
b) Refer to question 2b. What is the volume of a circular cone of radius r and height h whose top point is not located directly over the center of the base but is shifted k units to one side?
c) Give both an intuitive and a mathematical argument to support your answer to part b).

4. Derive the volumes for the following solids:

- a) a right pyramid whose altitude is h and whose base is a square with sides of length a ;
b) the water which is two inches deep in a hemispherical basin of radius one foot;
c) a solid object with cross sections being squares of side length $s(x) = \sqrt{\sin x}$;

5. Suppose you have a tent which is supported by two flexible aluminum poles which run along the ceiling of the tent from one corner to the opposite corner of a square tent floor, crossing at the highest point of the roof. Assume that the shape that the poles make is a parabola.
- If the tent has a square base 6 feet wide and is 6 feet tall at the highest point, find an equation for the parabola formed by the poles. (Be sure to identify what the variables you are using mean.)
 - What shape are all of the horizontal cross-sections of such a tent?
 - What is the area of the cross-section 4 feet above the ground?
 - What is the area of the cross-section z feet above the ground?
 - Set up an integral which equals the volume of the tent and evaluate the integral.