WORKSHEET 2

1. a) Give the definition of a definite integral.

   b) State the Fundamental Theorem of Calculus.

   c) What is wrong with the following argument?

$$\int_{-1}^{2} \frac{dt}{t^2} = \left.\frac{-1}{t}\right|_{-1}^{2} = -\frac{1}{2} - \left(-\frac{1}{-1}\right) = -\frac{1}{2} - 1 = -\frac{3}{2}$$

2. Determine whether or not the definite integral of the following functions over the corresponding intervals exist. If not, state why not. If so, evaluate the integral.

   a) $f(x) = e^x \qquad [0, 2]$

   b) $f(x) = e^x \qquad (-\infty, 0]$

   c) $f(x) = \dfrac{1}{x^2} \qquad [-1, 1]$

   d) $f(x) = 5x^5 - 2\sin x \qquad [-1, 1]$

   e) $f(x) = \begin{cases} 1 & \text{if } x \text{ is in } [0,1],\ [2.3, 2.5], \text{ or } [3, 3.6]; \\ \frac{1}{2} & \text{otherwise.} \end{cases} \qquad [-1, 5]$

   f) $f(x) = \begin{cases} 0 & \text{if } x \text{ is rational;} \\ 1 & \text{if } x \text{ is irrational.} \end{cases} \qquad [0, 1]$

   g) $f(x) = \begin{cases} x^2 + 1 & x \neq 0; \\ 0 & \text{x=0.} \end{cases} \qquad [-2, 2]$

   h) $f(x) = \ln x \qquad [0, 1]$

3. We are given a differentiable, odd function $f(x)$ defined on $[-3, 3]$ which has zeros at $x = -2$, $x = 0$, and $x = 2$ (nowhere else) and critical points at $x = -1$ and $x = 1$ (nowhere else). Also, we know that $f(-1) = 1$. Define a new function F on $[-3, 3]$ by the formula

$$F(x) = \int_{-2}^{x} f(t)\, dt.$$

   a) Sketch a rough graph of $f(x)$.

   b) Find the values of $F(-2)$, $F(2)$, and an upper and lower bound on $F(0)$.

   c) Find the critical points and inflection points of $F(x)$ on $[-3, 3]$.

   d) Sketch a rough graph of $F(x)$.

   e) Interpret the points found in (c) in terms of the graphs of $f(x)$ and $F(x)$.

4. A cake has dimensions $15'' \times 15'' \times 3''$. It is frosted on the sides and top. How can it be divided into 5 pieces so that each piece has the same amount of cake and the same amount of frosting? What if the dimensions are changed? What about 6 pieces? 7 pieces? n pieces?

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